CONTINUUM FIELD MODEL OF STREET CANYON: NUMERICAL EXAMPLES. PART 2

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Continuum field control model of a street canyon is considered. The six separate monocriterial optimal control problems consist of minimization of functionals of the total travel time, of global emissions of pollutants, and of global concentrations of pollutants, both in the studied street canyon, and in its two nearest neighbour substitute canyons, respectively. The six optimization problems for the functionals are solved numerically. General traffic control issues are inferred. The discretization method, comparison with experiment, mathematical issues, and programming issues, are presented.

1. INTRODUCTION

The present article is a continuation of [1]. The notation of [1] will be used. Now, we will solve numerically six separate monocriterial optimization problems **O1-O6**. We assumed the data from real street canyon of the Krasiński Avenue in Cracow [2]. We solve the set of nonlinear partial differential equations **E1-E8** with given boundary **B0-B8**, and the initial conditions **C0-C8**, and sources **D0-D2**, by the finite difference method, using the C language programme written by the author. We solve this set in the cuboid Ω, starting from initial conditions, and we iterate it over the time period using the direct finite difference method taking into account the boundary **B0-B8** conditions, and the sources **D0-D2**, and initial conditions **C0-C8**, at each time step. The functionals in **F1-F6** are iterated with the same steps as the equations **E1-E8** are iterated. The first time derivative is approximated by the first differential quotient using forward two-point first difference in the direction of time coordinate, whereas the spatial first derivatives

are approximated by the first differential quotients using central three-point first differences in the directions of spatial coordinates. In general, the numerical results are in good agreement with the measured data from [2].

2. GENERAL TRAFFIC MANAGEMENT INFERENCES

On the basis of the performed simulations, we deduce the traffic management inferences. In general, some vehicular traffic and hydrodynamical parameters influence the solutions of optimization problems O1-O6, but not all of them.

R1. The direction of velocity of air mixture is important. The optimal pollutant concentrations for both the single canyon and the canyon with its two substitute nearest neighbour canyons, O3, O6, are the lowest ones if the velocity components of the boundary and the initial value problems are equal to zero; further, they are grater if the velocity has only nonzero vertical component v_z; then, they are greater if velocity has only nonzero x-component v_x; further, they are again greater if the velocity has two nonzero components $(0, v_y, v_z)$; next, they are again greater if the velocity has three nonzero components (v_x, v_y, v_z); finally, they are the greatest if the velocity has only nonzero y-component v_y (compare Tables 1, 2). From Tables 1, 2, we infer that in case R1.6, the optimal

Table 1. Optimal solutions of problems O3, O6, for different velocities of mixture at temperature TH = 293.16 [K].

CASE	R1.1	R1.2	R1.3	R1.4
UNIFORM	0	0	0	0
yL [m]	0	0	0	0
yR [m]	0	0	0	0
VX [$\frac{m}{s}$]	1	0	0	0
VY [$\frac{m}{s}$]	0	1	0	1
VZ [$\frac{m}{s}$]	0	0	1	1
LON	1	1	1	1
RON	1	1	1	1
$C_{1,C}^*$ [s]	4.500000e+01	4.500000e+01	4.500000e+01	4.500000e+01
$C_{2,C}^*$ [s]	6.000000e+01	6.000000e+01	6.000000e+01	6.000000e+01
$g_{1,C}^*$ [s]	1.000000e+01	1.000000e+01	1.000000e+01	1.000000e+01
$g_{2,C}^*$ [s]	4.000000e+01	4.000000e+01	4.000000e+01	4.000000e+01
F_C^* [s]	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
J_C^* [kg·s]	8.017253e+05	1.342831e+06	7.515981e+05	1.301977e+06
$C_{1,C,ext}^*$ [s]	3.000000e+01	3.000000e+01	3.000000e+01	3.000000e+01
$C_{3,C,ext}^*$ [s]	4.500000e+01	4.500000e+01	3.000000e+01	4.500000e+01
$g_{1,C,ext}^*$ [s]	1.000000e+01	1.000000e+01	1.000000e+01	1.000000e+01
$g_{2,C,ext}^*$ [s]	2.500000e+01	2.500000e+01	2.500000e+01	2.500000e+01
$F_{C,ext}^*$ [s]	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
$J_{C,ext}^*$ [kg·s]	8.027054e+05	1.343811e+06	7.525782e+05	1.302958e+06

green time $g_{2,C}^*$ is different from other cases. It means that velocity of the mixture influences optimal vectors of control. The values of traffic parameters for Tables 1, 2, are given in Table 3, and the parameters of simulation are presented in Table 4.

Table 2. Optimal solutions of problems O3, O6, for different velocities of mixture at temperature TH = 293.16 [K] (continuation of Table 1).

CASE	R1.5	R1.6
UNIFORM	0	0
yL [m]	0	0
yR [m]	0	0
VX [$\frac{m}{s}$]	0	1
VY [$\frac{m}{s}$]	0	1
VZ [$\frac{m}{s}$]	0	1
LON	1	1
RON	1	1
$C_{1,C}^*$ [s]	4.500000e+01	4.500000e+01
$C_{2,C}^*$ [s]	6.000000e+01	6.000000e+01
$g_{1,C}^*$ [s]	1.000000e+01	1.000000e+01
$g_{2,C}^*$ [s]	4.000000e+01	3.250000e+01
F_C^* [s]	0.000000e+00	0.000000e+00
J_C^* [hg·s]	7.515922e+05	1.332875e+06
$C_{1,C,\text{ext}}^*$ [s]	3.000000e+01	3.000000e+01
$C_{2,C,\text{ext}}^*$ [s]	4.500000e+01	4.500000e+01
$g_{1,C,\text{ext}}^*$ [s]	1.000000e+01	1.000000e+01
$g_{2,C,\text{ext}}^*$ [s]	2.500000e+01	2.500000e+01
$F_{C,\text{ext}}^*$ [s]	0.000000e+00	0.000000e+00
$J_{C,\text{ext}}^*$ [kg· s]	7.525723e+05	1.333855e+06

Table 3. Values of maximum free flow speed, jam, saturation, threshold, green, and red vehicular densities for $VT = 4$ types of vehicles for UNIFORM = 0 for Tables 1, 2, 5, 6.

vt	1	2	3	4
$w_{vt,f}$ [$\frac{m}{s}$]	6.705500e+00	8.046600e+00	9.387700e+00	1.072880e+01
$k_{vt,\text{jam}}$ [$\frac{veh}{m}$]	5.435000e−02	6.522000e−02	7.609000e−02	8.696000e−02
$k_{vt,\text{sat}}$ [$\frac{veh}{m}$]	4.891500e−02	5.869800e−02	6.848100e−02	7.826400e−02
$k_{vt,\text{threshold}}$ [$\frac{veh}{m}$]	2.717500e−02	3.261000e−02	3.804500e−02	4.348000e−02
$k_{vt,\text{GREEN}}$ [$\frac{veh}{m}$]	1.500000e−02	1.800000e−02	2.100000e−02	2.400000e−02
$k_{vt,\text{RED}}$ [$\frac{veh}{m}$]	3.000000e−03	3.600000e−03	4.200000e−03	4.800000e−03

R2. The optimal values for cases **O1, O4**, for cases **O2, O5**, and for cases **O3, O6**, decrease with "uniformization" of the vehicles, when we pass from nonuniform vehicles UNIFORM = 0 to uniform ones UNIFORM = 1. It is a result of reduction of the number of vehicles moving in the canyon. For UNIFORM = 1

the values of maximum free flow speed, jam, saturation, threshold, green, and red densities reach the minimal values [3].

Table 4. Values of simulation parameters for Tables 1, 2, 5, 6, and for CASE = R9.1 of Table 7.

VT	4
n_L	2
n_R	2
$CT = N_E - 1$	3
a [m]	4.680000e+02
b [m]	4.400000e+01
c [m]	2.000000e+01
T_S [s]	6.000000e+01
δ_x [m]	4.680000e+01
δ_y [m]	8.800000e+00
δ_z [m]	4.000000e+00
δ_t [s]	4.800000e+00
δ_{C_1} [s]	1.500000e+01
δ_{C_2} [s]	1.500000e+01
δ_{g_1} [s]	7.500000e+00
δ_{g_2} [s]	7.500000e+00
δ_F [s]	7.500000e+00
$C_{1,min}$ [s]	3.000000e+01
$C_{2,min}$ [s]	3.000000e+01
$C_{1,min}$ [s]	1.000000e+01
$C_{2,min}$ [s]	1.000000e+01
F_{min} [s]	0.000000e+00
$C_{1,max}$ [s]	6.000000e+01
$C_{2,max}$ [s]	6.000000e+01
$g_{1,orth}$ [s]	1.500000e+01
$g_{2,orth}$ [s]	1.500000e+01

R3. The long vehicular queues decrease the total travel times **O1, O4**, and they increase both the optimal emissions **O2, O5**, and optimal concentrations of pollutants **O3, O6** [3]. The decrement of total travel times **O1, O4**, with long vehicular queues is a result of clustering of the vehicles.

R4. The constant of temperature scale TH does not differentiate the values of optimal concentrations of pollutants **O3, O6** in the temperature range near standard temperature and pressure STP conditions. However, it diminishes them even hundredfold for very high temperatures (compare Table 5). The values of traffic parameters for Table [5] are given in Table 3, and the parameters of simulation are presented in Table 4.

Table 5. Optimal solutions of problems O1, O2, O3, O4, O5, O6, with different temperature scales TH.

CASE	R4.1	R4.2	R4.3	R4.4
TH [K]	293.16	273.16	1.000000e+10	1.000000e+08
UNIFORM	0	0	0	0
yL [m]	0	0	0	0
yR [m]	0	0	0	0
VX [$\frac{m}{s}$]	1	1	1	1
LON	1	1	1	1
RON	1	1	1	1
$C^*_{1,\text{TTT}}$ [s]	6.000000e+01	6.000000e+01	6.000000e+01	6.000000e+01
$C^*_{2,\text{TTT}}$ [s]	6.000000e+01	6.000000e+01	6.000000e+01	6.000000e+01
$g^*_{1,\text{TTT}}$ [s]	4.000000e+01	4.000000e+01	4.000000e+01	4.000000e+01
$g^*_{2,\text{TTT}}$ [s]	4.000000e+01	4.000000e+01	4.000000e+01	4.000000e+01
F^*_{TTT} [s]	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
J^*_{TTT} [veh· s]	1.039254e+05	1.039254e+05	1.039254e+05	1.039254e+05
$C^*_{1,\text{E}}$ [s]	4.500000e+01	4.500000e+01	4.500000e+01	4.500000e+01
$C^*_{2,\text{E}}$ [s]	6.000000e+01	6.000000e+01	6.000000e+01	6.000000e+01
$g^*_{1,\text{E}}$ [s]	1.000000e+01	1.000000e+01	1.000000e+01	1.000000e+01
$g^*_{2,\text{E}}$ [s]	3.250000e+01	3.250000e+01	3.250000e+01	3.250000e+01
F^*_{E} [s]	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
J^*_{E} [kg]	7.434836e−01	7.434836e−01	7.434836e−01	7.434836e−01
$C^*_{1,\text{C}}$ [s]	4.500000e+01	4.500000e+01	4.500000e+01	4.500000e+01
$C^*_{2,\text{C}}$ [s]	6.000000e+01	6.000000e+01	6.000000e+01	6.000000e+01
$g^*_{1,\text{C}}$ [s]	1.000000e+01	1.000000e+01	1.000000e+01	1.000000e+01
$g^*_{2,\text{C}}$ [s]	4.000000e+01	4.000000e+01	4.000000e+01	4.000000e+01
F^*_{C} [s]	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
J^*_{C} [kg· s]	8.017253e+05	8.017253e+05	1.375833e+03	7.068254e+05
$C^*_{1,\text{TTT,ext}}$ [s]	6.000000e+01	6.000000e+01	6.000000e+01	6.000000e+01
$C^*_{2,\text{TTT,ext}}$ [s]	6.000000e+01	6.000000e+01	6.000000e+01	6.000000e+01
$g^*_{1,\text{TTT,ext}}$ [s]	4.000000e+01	4.000000e+01	4.000000e+01	4.000000e+01
$g^*_{2,\text{TTT,ext}}$ [s]	4.000000e+01	4.000000e+01	4.000000e+01	4.000000e+01
$F^*_{\text{TTT,ext}}$ [s]	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
$J^*_{\text{TTT,ext}}$ [veh· s]	1.510210e+05	1.510210e+05	1.510210e+05	1.510210e+05
$C^*_{1,\text{E,ext}}$ [s]	3.000000e+01	3.000000e+01	3.000000e+01	3.000000e+01
$C^*_{2,\text{E,ext}}$ [s]	4.500000e+01	4.500000e+01	4.500000e+01	4.500000e+01
$g^*_{1,\text{E,ext}}$ [s]	1.000000e+01	1.000000e+01	1.000000e+01	1.000000e+01
$g^*_{2,\text{E,ext}}$ [s]	2.500000e+01	2.500000e+01	2.500000e+01	2.500000e+01
$F^*_{\text{E,ext}}$ [s]	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
$J^*_{\text{E,ext}}$ [kg]	2.956799e+00	2.956799e+00	2.956799e+00	2.956799e+00
$C^*_{1,\text{C,ext}}$ [s]	3.000000e+01	3.000000e+01	3.000000e+01	3.000000e+01
$C^*_{2,\text{C,ext}}$ [s]	4.500000e+01	4.500000e+01	4.500000e+01	4.500000e+01
$g^*_{1,\text{C,ext}}$ [s]	1.000000e+01	1.000000e+01	1.000000e+01	1.000000e+01
$g^*_{2,\text{C,ext}}$ [s]	2.500000e+01	2.500000e+01	2.500000e+01	2.500000e+01
$F^*_{\text{C,ext}}$ [s]	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
$J^*_{\text{C,ext}}$ [10^5kg· s]	8.027054e+05	8.027054e+05	2.355926e+03	7.078055e+05

R5. The functional form of initial and boundary conditions affects the optima. If they are constant, then optimal concentrations **O3, O6**, are two times higher than in the case when they are changing exponentially in space in three dimensions (compare Table 6). The index FORM = CONSTANT corresponds to constant initial and boundary conditions, whereas FORM = EXPONENTIAL corresponds to the spatially exponentially changing initial and boundary conditions. For the latter case, we chose the following functional form of initial conditions: the temperature is equal to $T_0(x, y, z, t) = \exp\left(-\dfrac{x}{a} - \dfrac{y}{b} - \dfrac{z}{c}\right)$ TH, the only non-zero coordinates of velocities of mixture are the x-coordinates and they are equal to $v_{x,0}(x, y, z, t) = \exp\left(-\dfrac{x}{a} - \dfrac{y}{b} - \dfrac{z}{c}\right)$ VX, the density is equal to $\rho_0(x, y, z, t) = \exp\left(-\dfrac{x}{a} - \dfrac{y}{b} - \dfrac{z}{c}\right)$ ρ_{STP}, the concentrations are equal to $c_{i,0}(x, y, z, t) = \exp\left(-\dfrac{x}{a} - \dfrac{y}{b} - \dfrac{z}{c}\right)$ $c_{i,STP}$, and the pressure is equal to $p_0(x, y, z, t) = \exp\left(-\dfrac{x}{a} - \dfrac{y}{b} - \dfrac{z}{c}\right)$ p_{STP}. The boundary conditions **B0-B8** are similar. The values of traffic parameters for Table 6 are given in Table 3, and the parameters of simulation are presented in Table 4.

R6. The presence of vehicles on both the left and right lanes is important. The optimal total travel times and emissions are halved in absence of vehicles on the left or right lanes when compared to situation when they circulate on both the left and right lanes [3].

R7. The values of saturation, arrival, or jam vehicular density, and of vehicular free flow velocities also affect the optima **O1-O6** [3].

R8. The assumption of energy conservation equation, of the thermodiffusion effect, of the chemical potential and of the Grand Canonical ensemble, as well as of influence of gravity on intrinsic energy and on the chemical potential, drastically changes the optimal concentrations **O3, O6**, towards the measured ones [3, 4, 5].

R9. The value of time of simulation and of discretization in time considerably affects the optima (compare Table 7 and [3]). The values of optimal solutions **O1-O6**, increase from tenfold to hundredfold. Also the optimal vectors (5-tuples) of control for **O1-O6**, change their values. It is a result of cumulative effect of length of the period of simulation T_S on integral functionals **F1-F6**. The values of traffic parameters for CASE = R9.1, R9.2, are given in Table 8. The parameters of simulation for CASE = R9.1 are presented in Table 4, for CASE = R9.2 in Table 9.

We use the following notation: yL = 0/yR = 0 means that the lengths of all queues on the left/right lanes at the beginning were equal to zero, VX = 0 is meant for zero initial and boundary velocity of mixture, while VX = 1 is put for

Table 6. Optimal solutions of problems O1, O2, O3, O4, O5, O6, with different initial and boundary conditions for TH = 293.16 [K].

CASE	R5.1	R5.2
FORM	CONSTANT	EXPONENTIAL
UNIFORM	0	0
yL [m]	0	0
yR [m]	0	0
VX [$\frac{m}{s}$]	1	1
LON	1	1
RON	1	1
$C^*_{1,TTT}$ [s]	6.000000e+01	6.000000e+01
$C^*_{2,TTT}$ [s]	6.000000e+01	6.000000e+01
$g^*_{1,TTT}$ [s]	4.000000e+01	4.000000e+01
$g^*_{2,TTT}$ [s]	4.000000e+01	4.000000e+01
F^*_{TTT} [s]	0.000000e+00	0.000000e+00
J^*_{TTT} [veh· s]	1.039254e+05	1.039254e+05
$C^*_{1,E}$ [s]	4.500000e+01	4.500000e+01
$C^*_{2,E}$ [s]	6.000000e+01	6.000000e+01
$g^*_{1,E}$ [s]	1.000000e+01	1.000000e+01
$g^*_{2,E}$ [s]	3.250000e+01	3.250000e+01
F^*_{E} [s]	0.000000e+00	0.000000e+00
J^*_{E} [kg]	7.434836e−01	7.434836e−01
$C^*_{1,C}$ [s]	4.500000e+01	4.500000e+01
$C^*_{2,C}$ [s]	6.000000e+01	6.000000e+01
$g^*_{1,C}$ [s]	1.000000e+01	1.000000e+01
$g^*_{2,C}$ [s]	4.000000e+01	4.000000e+01
F^*_{C} [s]	0.000000e+00	0.000000e+00
J^*_{C} [10^5kg· s]	8.017253	4.934016
$C^*_{1,TTT,ext}$ [s]	6.000000e+01	6.000000e+01
$C^*_{2,TTT,ext}$ [s]	6.000000e+01	6.000000e+01
$g^*_{1,TTT,ext}$ [s]	4.000000e+01	4.000000e+01
$g^*_{2,TTT,ext}$ [s]	4.000000e+01	4.000000e+01
$F^*_{TTT,ext}$ [s]	0.000000e+00	0.000000e+00
$J^*_{TTT,ext}$ [veh· s]	1.510210e+05	1.510210e+05
$C^*_{1,E,ext}$ [s]	3.000000e+01	3.000000e+01
$C^*_{2,E,ext}$ [s]	4.500000e+01	4.500000e+01
$g^*_{1,E,ext}$ [s]	1.000000e+01	1.000000e+01
$g^*_{2,E,ext}$ [s]	2.500000e+01	2.500000e+01
$F^*_{E,ext}$ [s]	0.000000e+00	0.000000e+01
$J^*_{E,ext}$ [kg]	2.956799e+00	2.956799e+00
$C^*_{1,C,ext}$ [s]	3.000000e+01	3.000000e+01
$C^*_{2,C,ext}$ [s]	4.500000e+01	4.500000e+01
$g^*_{1,C,ext}$ [s]	1.000000e+01	1.000000e+01
$g^*_{2,C,ext}$ [s]	2.500000e+01	2.500000e+01
$F^*_{C,ext}$ [s]	0.000000e+00	0.000000e+00
$J^*_{C,ext}$ [10^5kg· s]	8.027054	4.943817

Table 7. Optimal solutions of problems O1, O2, O3, O4, O5, O6, with different times of simulation T_S.

CASE	R9.1	R9.2
T_S [s]	6.000000e+01	1.800000e+03
UNIFORM	0	0
yL [m]	0	0
yR [m]	0	0
VX [$\frac{m}{s}$]	1	1
LON	1	1
RON	1	1
$C^*_{1,\mathrm{TTT}}$ [s]	6.000000e+01	3.000000e+01
$C^*_{2,\mathrm{TTT}}$ [s]	6.000000e+01	3.000000e+01
$g^*_{1,\mathrm{TTT}}$ [s]	4.000000e+01	1.000000e+01
$g^*_{2,\mathrm{TTT}}$ [s]	2.500000e+01	1.000000e+01
F^*_{TTT} [s]	0.000000e+00	0.000000e+00
J^*_{TTT} [veh· s]	4.352085e+04	6.250812e+05
$C^*_{1,\mathrm{E}}$ [s]	4.500000e+01	1.425000e+02
$C^*_{2,\mathrm{E}}$ [s]	6.000000e+01	1.425000e+02
$g^*_{1,\mathrm{E}}$ [s]	1.000000e+01	1.000000e+01
$g^*_{2,\mathrm{E}}$ [s]	4.000000e+01	4.750000e+01
F^*_{E} [s]	7.5000000e+00	9.375000e+01
J^*_{E} [kg]	5.617436e−02	1.947124e+01
$C^*_{1,\mathrm{C}}$ [s]	4.500000e+01	1.050000e+02
$C^*_{2,\mathrm{C}}$ [s]	6.000000e+01	1.425000e+02
$g^*_{1,\mathrm{C}}$ [s]	1.000000e+01	1.000000e+01
$g^*_{2,\mathrm{C}}$ [s]	4.000000e+01	1.000000e+01
F^*_{C} [s]	0.000000e+00	0.000000e+00
J^*_{C} [kg· s]	8.017248e+05	9.103424e+07
$C^*_{1,\mathrm{TTT,ext}}$ [s]	6.000000e+01	3.000000e+01
$C^*_{2,\mathrm{TTT,ext}}$ [s]	4.500000e+01	3.000000e+01
$g^*_{1,\mathrm{TTT,ext}}$ [s]	4.000000e+01	1.000000e+01
$g^*_{2,\mathrm{TTT,ext}}$ [s]	2.500000e+01	1.000000e+01
$F^*_{\mathrm{TTT,ext}}$ [s]	0.000000e+00	0.000000e+00
$J^*_{\mathrm{TTT,ext}}$ [veh· s]	5.080032e+04	6.405641e+05
$C^*_{1,\mathrm{E,ext}}$ [s]	3.000000e+01	1.425000e+02
$C^*_{2,\mathrm{E,ext}}$ [s]	6.000000e+01	1.425000e+02
$g^*_{1,\mathrm{E,ext}}$ [s]	1.000000e+01	1.000000e+01
$g^*_{2,\mathrm{E,ext}}$ [s]	4.000000e+01	1.225000e+02
$F^*_{\mathrm{E,ext}}$ [s]	7.500000e+00	9.375000e+01
$J^*_{\mathrm{E,ext}}$ [kg]	2.197456e−01	2.120986e+01
$C^*_{1,\mathrm{C,ext}}$ [s]	3.000000e+01	1.050000e+02
$C^*_{2,C.ext}$ [s]	6.000000e+01	6.750000e+01
$g^*_{1,\mathrm{C,ext}}$ [s]	1.000000e+01	1.000000e+01
$g^*_{2,\mathrm{C,ext}}$ [s]	4.000000e+01	1.000000e+01
$F^*_{\mathrm{C,ext}}$ [s]	0.000000e+00	1.875000e+01
$J^*_{\mathrm{C,ext}}$ [kg· s]	8.027048e+05	9.103798e+07

Table 8. Values of maximum free flow speed, jam, saturation, threshold, green, and red vehicular densities for $VT = 4$ types of vehicles for UNIFORM $= 0$ for CASE $=$ R9.1, R9.2, of Table 7.

vt	1	2	3	4
$w_{vt,f} \left[\frac{\text{m}}{\text{s}}\right]$	1.500000e+01	1.350000e+01	1.200000e+01	1.050000e+01
$k_{vt,\text{jam}} \left[\frac{\text{veh}}{\text{m}}\right]$	6.670000e$-$02	2.001000e$-$03	1.334000e$-$03	6.670000e$-$04
$k_{vt,\text{sat}} \left[\frac{\text{veh}}{\text{m}}\right]$	6.003000e$-$02	1.800900e$-$03	1.200600e$-$03	6.003000e$-$04
$k_{vt,\text{threshold}} \left[\frac{\text{veh}}{\text{m}}\right]$	3.335000e$-$02	1.000500e$-$03	6.670000e$-$04	3.335000e$-$04
$k_{vt,\text{GREEN}} \left[\frac{\text{veh}}{\text{m}}\right]$	6.670000e$-$02	2.001000e$-$03	1.334000e$-$03	6.670000e$-$04
$k_{vt,\text{RED}} \left[\frac{\text{veh}}{\text{m}}\right]$	1.334000e$-$02	4.002000e$-$04	2.668000e$-$04	1.334000e$-$04

Table 9. Values of simulation parameters for CASE $=$ R9.2 of Table 7.

VT	4
n_L	2
n_B	2
$CT = N_E - 1$	3
a [m]	4.680000e+02
b [m]	4.400000e+01
c [m]	2.000000e+01
T_S [s]	1.800000e+03
δ_x [m]	4.680000e+01
δ_y [m]	8.800000e+00
δ_z [m]	4.000000e+00
δ_t [s]	1.800000e+01
δ_{C_1} [s]	3.750000e+01
δ_{C_2} [s]	3.750000e+01
δ_{g_1} [s]	1.875000e+01
δ_{g_2} [s]	1.875000e+01
δ_F [s]	1.875000e+01
$C_{1,\text{min}}$ [s]	3.000000e+01
$C_{2,\text{min}}$ [s]	3.000000e+01
$C_{1,\text{min}}$ [s]	1.000000e+01
$C_{2,\text{min}}$ [s]	1.000000e+01
F_{min} [s]	0.000000e+00
$C_{1,\text{max}}$ [s]	1.500000e+02
$C_{2,\text{max}}$ [s]	1.500000e+02
$g_{1,\text{orth}}$ [s]	1.500000e+01
$g_{2,\text{orth}}$ [s]	1.500000e+01

the velocity of $1 \left[\frac{\text{m}}{\text{s}}\right]$. The same holds for VY and VZ. If VX $\neq 0$, then the left lanes are leeward and the right ones windward. If there are no vehicles on the

left or right lanes, then LON = 0 or RON = 0, respectively. UNIFORM = 0 stands for non-homogeneous (different) values of maximum free flow speed $w_{vt,f}$, jam $k_{vt,\text{jam}}$, saturation $\overline{k}_{vt,\text{sat}}$, threshold $k_{vt,\text{threshold}}$, green $k_{vt,\text{GREEN}}$, and red $k_{vt,\text{RED}}$, vehicular densities for $VT = 4$ types of vehicles: passenger cars, 8-ton, 12-ton, and 16-ton trucks, where threshold vehicular densities $k_{vt,\text{threshold}}$ are the numerical values for which we assume in numerical simulations that there is a vehicle on the given lane. We also assume that these values are the same for both the left and right lanes. $\delta_x, \delta_y, \delta_z, \delta_t$, are the steps in x, y, z, t, directions, respectively. $\delta_{C_1}, \delta_{g_1}, \delta_{C_2}, \delta_{g_2}, \delta_F$, are the steps in C_1, g_1, C_2, g_2, F, directions, respectively. In the boundary and initial conditions **B0a, B0b, B0c, C0**, we assume that the temperature is equal to a given constant TH.

3. DISCRETIZATION SCHEME

In order to solve numerically the set of nonlinear partial differential equations **E1-E8**, one uses the apparatus of finite differences. Firstly, we discretize the points of domain Σ in the standard way:

$$(3.1) \qquad (x, y, z, t) \simeq (x_i, y_j, z_k, t_l), \quad i = 0, \cdots, N_x, \quad j = 0, \cdots, N_y,$$
$$k = 0, \cdots, N_z, \quad l = 0, \cdots, N_t,$$

$$(3.2) \qquad x_i = x_0 + \frac{i-1}{N_x}(x_{N_x} - x_0),$$

$$(3.3) \qquad y_i = y_0 + \frac{j-1}{N_y}(y_{N_y} - y_0),$$

$$(3.4) \qquad z_k = z_0 + \frac{k-1}{N_z}(z_{N_z} - z_0),$$

$$(3.5) \qquad t_l = t_0 + \frac{l-1}{N_t}(t_{N_t} - t_0).$$

Secondly, the value of given function f at point (x, y, z, t) of its domain Σ is approximated by:

$$(3.6) \qquad f(x, y, z, t) \simeq f(x_i, y_i, z_k, t_l) \equiv f_{i,j,k,l}.$$

Thirdly, the first finite differences in spatial and temporal directions:

$$(3.7) \qquad \Delta_x f_{i,j,k,l} = \frac{1}{2}(f_{i+1,j,k,l} - f_{i-1,j,k,l}),$$

$$(3.8) \qquad \Delta_y f_{i,j,k,l} = \frac{1}{2}(f_{i,j+1,k,l} - f_{i,j-1,k,l}),$$

$$(3.9) \qquad \Delta_z f_{i,j,k,l} = \frac{1}{2}(f_{i,j,k+1,l} - f_{i,j,k-1,l}),$$

$$(3.10) \qquad \Delta_t f_{i,j,k,l} = f_{i,j,k,l+1} - f_{i,j,k,l},$$

allow to approximate the first partial derivatives:

$$(3.11) \qquad \frac{\partial f(x,y,z,t)}{\partial x} \simeq \frac{\Delta_x f_{i,j,k,l}}{\Delta x},$$

$$(3.12) \qquad \frac{\partial f(x,y,z,t)}{\partial y} \simeq \frac{\Delta_y f_{i,j,k,l}}{\Delta y},$$

$$(3.13) \qquad \frac{\partial f(x,y,z,t)}{\partial z} \simeq \frac{\Delta_z f_{i,j,k,l}}{\Delta z},$$

$$(3.14) \qquad \frac{\partial f(x,y,z,t)}{\partial t} \simeq \frac{\Delta_t f_{i,j,k,l}}{\Delta t}.$$

Since the vector and scalar partial differential equations **E1-E8** are of the following form:

$$(3.15) \qquad \frac{\partial f^\alpha(x,y,z,t)}{\partial t} = F^\alpha\left(x,y,z,t,f^\alpha(x,y,z,t), \frac{\partial f^\alpha(x,y,z,t)}{\partial x_m},\right.$$
$$\left. \frac{\partial^2 f^\alpha(x,y,z,t)}{\partial x_m \partial x_n}, f^\beta(x,y,z,t)\right),$$

hence we approximate them as follows:

$$(3.16) \qquad \frac{\Delta_t f^\alpha_{i,j,k,l}}{\Delta t} = F^\alpha\left(x_i, y_j, z_k, t_l, f^\alpha_{i,j,k,l}, \frac{\Delta_{x_m} f^\alpha_{i,j,k,l}}{\Delta x_m},\right.$$
$$\left. \frac{\Delta_{x_m}\Delta_{x_n} f^\alpha_{i,j,k,l}}{\Delta x_m \Delta x_n}, f^\beta_{i,j,k,l}\right).$$

Hence, we can rewrite it in following forward scheme:

$$(3.17) \qquad f^\alpha_{i,j,k,l+1} = f^\alpha_{i,j,k,l} + \Delta t \cdot F^\alpha\left(x_i, y_j, z_k, t_l, f^\alpha_{i,j,k,l}, \frac{\Delta_{x_m} f^\alpha_{i,j,k,l}}{\Delta x_m},\right.$$
$$\left. \frac{\Delta_{x_m}\Delta_{x_n} f^\alpha_{i,j,k,l}}{\Delta x_m \Delta x_n}, f^\beta_{i,j,k,l}\right).$$

We give now the examples of dicretized equations. The discretization of continuity equation **E2** reads:

$$(3.18) \quad \rho_{i,j,k,l+1} = \rho_{i,j,k,l} - \frac{\Delta t}{2\Delta x} v^x_{i,j,k,l}(\rho_{i+1,j,k,l} - \rho_{i-1,j,k,l})$$

$$- \frac{\Delta t}{2\Delta y} v^y_{i,j,k,l}(\rho_{i,j+1,k,l} - \rho_{i,j-1,k,l}) - \frac{\Delta t}{2\Delta z} v^z_{i,j,k,l}(\rho_{i,j,k+1,l} - \rho_{i,j,k-1,l})$$

$$+ \Delta t \frac{T_S}{\rho_0} S_{i,j,k,l} - \frac{\Delta t}{2\Delta x} \rho_{i,j,k,l}(\rho_{i+1,j,k,l} - \rho_{i-1,j,k,l})$$

$$- \frac{\Delta t}{2\Delta y} \rho_{i,j,k,l}(\rho_{i,j+1,k,l} - \rho_{i,j-1,k,l}) - \frac{\Delta t}{2\Delta z} \rho_{i,j,k,l}(\rho_{i,j,k+1,l} - \rho_{i,j,k-1,l}).$$

For the x-component of Navier-Stokes equation **E1** we have:

$$(3.19) \quad v^x_{i,j,k,l+1} = v^x_{i,j,k,l} - \frac{\Delta t}{2\Delta x} v^x_{i,j,k,l}(v^x_{i+1,j,k,l} - v^x_{i-1,j,k,l})$$

$$- \frac{\Delta t}{2\Delta y} v^y_{i,j,k,l}(v^x_{i,j+1,k,l} - v^x_{i,j-1,k,l}) - \frac{\Delta t}{2\Delta z} v^z_{i,j,k,l}(v^x_{i,j,k+1,l} - v^x_{i,j,k-1,l})$$

$$- \Delta t \frac{T_S}{\rho_0} \frac{v^x_{i,j,k,l}}{\rho_{i,j,k,l}} S_{i,j,k,l} - \frac{\Delta t}{2\Delta x} BT_0 \frac{T_S^2}{a^2}(T_{i+1,j,k,l} - T_{i-1,j,k,l})$$

$$- \frac{\Delta t}{2\Delta x} BT_0 \frac{T_S^2}{a^2} \frac{T_{i,j,k,l}}{\rho_{i,j,k,l}}(\rho_{i+1,j,k,l} - \rho_{i-1,j,k,l})$$

$$+ \frac{\Delta t}{(\Delta x)^2} \frac{\eta T_S}{\rho_0 a^2} \frac{1}{\rho_{i,j,k,l}}(v^x_{i+1,j,k,l} - 2v^x_{i,j,k,l} + v^x_{i-1,j,k,l})$$

$$+ \frac{\Delta t}{(\Delta y)^2} \frac{\eta T_S}{\rho_0 b^2} \frac{1}{\rho_{i,j,k,l}}(v^x_{i,j+1,k,l} - 2v^x_{i,j,k,l} + v^x_{i,j-1,k,l})$$

$$+ \frac{\Delta t}{(\Delta z)^2} \frac{\eta T_S}{\rho_0 c^2} \frac{1}{\rho_{i,j,k,l}}(v^x_{i,j,k+1,l} - 2v^x_{i,j,k,l} + v^x_{i,j,k-1,l})$$

$$+ \frac{\Delta t}{(\Delta x)^2} \left(\xi + \frac{\eta}{3}\right) \frac{\eta T_S}{\rho_0 a^2} \frac{1}{\rho_{i,j,k,l}}(v^x_{i+1,j,k,l} - 2v^x_{i,j,k,l} + v^x_{i-1,j,k,l})$$

$$+ \frac{\Delta t}{4\Delta x \Delta y} \left(\xi + \frac{\eta}{3}\right) \frac{\eta T_S}{\rho_0 b^2} \frac{1}{\rho_{i,j,k,l}}(v^x_{i+1,j+1,k,l} - v^x_{i+1,j-1,k,l} - v^x_{i-1,j+1,k,l} + v^x_{i-1,j-1,k,l})$$

$$+ \frac{\Delta t}{4\Delta x \Delta z} \left(\xi + \frac{\eta}{3}\right) \frac{\eta T_S}{\rho_0 c^2} \frac{1}{\rho_{i,j,k,l}}(v^x_{i+1,j,k+1,l} - v^x_{i+1,j,k-1,l} - v^x_{i-1,j,k+1,l} + v^x_{i-1,j,k-1,l})$$

$$+ \Delta t \frac{g_0 T_S^2}{a} g^x.$$

Similarly for the y-component:

$$(3.20) \quad v^y_{i,j,k,l+1} = v^y_{i,j,k,l} - \frac{\Delta t}{2\Delta x} v^x_{i,j,k,l}(v^y_{i+1,j,k,l} - v^y_{i-1,j,k,l})$$

$$- \frac{\Delta t}{2\Delta y} v^y_{i,j,k,l}(v^y_{i,j+1,k,l} - v^y_{i,j-1,k,l}) - \frac{\Delta t}{2\Delta z} v^z_{i,j,k,l}(v^z_{i,j,k+1,l} - v^z_{i,j,k-1,l})$$

$$- \Delta t \frac{T_S}{\rho_0} \frac{v^y_{i,j,k,l}}{\rho_{i,j,k,l}} S_{i,j,k,l} - \frac{\Delta t}{2\Delta y} BT_0 \frac{T_S^2}{b^2}(T_{i,j+1,k,l} - T_{i,j-1,k,l})$$

$$- \frac{\Delta t}{2\Delta y} BT_0 \frac{T_S^2}{b^2} \frac{T_{i,j,k,l}}{\rho_{i,j,k,l}}(\rho_{i,j+1,k,l} - \rho_{i,j-1,k,l})$$

$$+ \frac{\Delta t}{(\Delta x)^2} \frac{\eta T_S}{\rho_0 a^2} \frac{1}{\rho_{i,j,k,l}}(v^y_{i+1,j,k,l} - 2v^y_{i,j,k,l} + v^y_{i-1,j,k,l})$$

$$+ \frac{\Delta t}{(\Delta y)^2} \frac{\eta T_S}{\rho_0 b^2} \frac{1}{\rho_{i,j,k,l}}(v^y_{i,j+1,k,l} - 2v^y_{i,j,k,l} + v^y_{i,j-1,k,l})$$

$$+ \frac{\Delta t}{(\Delta z)^2} \frac{\eta T_S}{\rho_0 c^2} \frac{1}{\rho_{i,j,k,l}}(v^y_{i,j,k+1,l} - 2v^y_{i,j,k,l} + v^y_{i,j,k-1,l})$$

$$+ \frac{\Delta t}{4\Delta x \Delta y}\left(\xi + \frac{\eta}{3}\right)\frac{\eta T_S}{\rho_0 b^2}\frac{1}{\rho_{i,j,k,l}}(v^x_{i+1,j+1,k,l} - v^x_{i+1,j-1,k,l} - v^x_{i-1,j+1,k,l} + v^x_{i-1,j-1,k,l})$$

$$+ \frac{\Delta t}{\Delta y^2}\left(\xi + \frac{\eta}{3}\right)\frac{\eta T_S}{\rho_0 b^2}\frac{1}{\rho_{i,j,k,l}}(v^y_{i+1,j+1,k,l} - v^y_{i+1,j-1,k,l} - v^y_{i-1,j+1,k,l} + v^y_{i-1,j-1,k,l})$$

$$+ \frac{\Delta t}{4\Delta y \Delta z}\left(\xi + \frac{\eta}{3}\right)\frac{\eta T_S}{\rho_0 b^2}\frac{1}{\rho_{i,j,k,l}}(v^x_{i,j+1,k+1,l} - v^x_{i,j+1,k-1,l} - v^x_{i,j-1,k+1,l} + v^x_{i,j-1,k-1,l})$$

$$+ \Delta t \frac{g_0 T_S^2}{b} g^y.$$

Finally, for the z-component:

$$(3.21) \quad v^z_{i,j,k,l+1} = v^z_{i,j,k,l} - \frac{\Delta t}{2\Delta x} v^x_{i,j,k,l}(v^z_{i+1,j,k,l} - v^z_{i-1,j,k,l})$$

$$\text{(3.21)} \atop \text{[cont.]} \qquad - \frac{\Delta t}{2\Delta y} v_{i,j,k,l}^{y} (v_{i,j+1,k,l}^{z} - v_{i,j-1,k,l}^{z}) - \frac{\Delta t}{2\Delta z} v_{i,j,k,l}^{z} (v_{i,j,k+1,l}^{z} - v_{i,j,k-1,l}^{z})$$

$$- \Delta t \frac{T_S}{\rho_0} \frac{v_{i,j,k,l}^{z}}{\rho_{i,j,k,l}} S_{i,j,k,l} - \frac{\Delta t}{2\Delta z} BT_0 \frac{T_S^2}{c^2} (T_{i,j,k+1,l} - T_{i,j,k-1,l})$$

$$- \frac{\Delta t}{2\Delta z} BT_0 \frac{T_S^2}{c^2} \frac{T_{i,j,k,l}}{\rho_{i,j,k,l}} (\rho_{i,j,k+1,l} - \rho_{i,j,k-1,l})$$

$$+ \frac{\Delta t}{(\Delta x)^2} \frac{\eta T_S}{\rho_0 a^2} \frac{1}{\rho_{i,j,k,l}} (v_{i+1,j,k,l}^{z} - 2v_{i,j,k,l}^{z} + v_{i-1,j,k,l}^{z})$$

$$+ \frac{\Delta t}{(\Delta y)^2} \frac{\eta T_S}{\rho_0 b^2} \frac{1}{\rho_{i,j,k,l}} (v_{i,j+1,k,l}^{z} - 2v_{i,j,k,l}^{z} + v_{i,j-1,k,l}^{z})$$

$$+ \frac{\Delta t}{(\Delta z)^2} \frac{\eta T_S}{\rho_0 c^2} \frac{1}{\rho_{i,j,k,l}} (v_{i,j,k+1,l}^{z} - 2v_{i,j,k,l}^{z} + v_{i,j,k-1,l}^{z})$$

$$+ \frac{\Delta t}{4\Delta x \Delta z} \left(\xi + \frac{\eta}{3} \right) \frac{\eta T_S}{\rho_0 c^2} \frac{1}{\rho_{i,j,k,l}} (v_{i+1,j,k+1,l}^{x} - v_{i+1,j,k-1,l}^{x} - v_{i-1,j,k+1,l}^{x} + v_{i-1,j,k-1,l}^{x})$$

$$+ \frac{\Delta t}{4\Delta y \Delta z} \left(\xi + \frac{\eta}{3} \right) \frac{\eta T_S}{\rho_0 b^2} \frac{1}{\rho_{i,j,k,l}} (v_{i,j+1,k+1,l}^{x} - v_{i,j,k-1,l}^{x} - v_{i,j,k+1,l}^{x} + v_{i,j-1,k-1,l}^{x})$$

$$+ \frac{\Delta t}{(\Delta z)^2} \left(\xi + \frac{\eta}{3} \right) \frac{\eta T_S}{\rho_0 c^2} \frac{1}{\rho_{i,j,k,l}} (v_{i,j,k+1,l}^{z} - 2v_{i,j,k,l}^{z} + v_{i,j,k-1,l}^{z}) + \Delta t \frac{g_0 T_S^2}{c} g^z.$$

4. METHOD OF COMPARISON

We briefly describe the method of comparison of the numerical results with the experimental ones. The comparison of simulations with the measured data, if authorized, is aimed at showing the correctness of the model (conservation of the rank of simulated parameters), and of presenting the correct direction of deviations (the deviations always tend to secure the direction). We transform both measurements and optima J_C^* **O3**, in order to compare them. The measured pollutant concentrations are multiplied by the volume of the canyon, and we obtain pollutant masses (we assume that the measured concentrations are the same in the entire canyon and in time). The pollutant masses are added up, assuming HC mass to be equal to zero (HC were not measured in Ref. [2]). The calculated optima J_C^* **O3** are divided by time of simulation T_S, giving approximation

of pollutant masses in every moment of simulation (we assume that J_C^* **O3** are homogeneous in time, in order to make possible the comparison with measurements). Thence, we have two comparable magnitudes. Also, only insufficient vehicular flow data are given in Ref. [2]. Neither the jam, saturation vehicular densities, nor the maximum vehicular speeds are present. However, the initial and boundary conditions of traffic flow **B5-B8, C5-C8** are not constant functions because they are parametrized by measured traffic parameters. The data are not instantaneous, but they are measured in intervals of hours. Moreover, the measured data are not accompanied by measurements of wind velocity, temperature, density. If the measurements had been performed instantly using more specialized equipment (e. g., lidar), and if they had been stored in computer facility, then, it would be possible to make more serious comparisons. The simulations were performed for the time period T = 60 [s], whereas the measurement were averaged over the time period T = 1800 [s]. The integral functionals **F3, F6**, are cumulative with respect to the time period.

5. COMPUTER SIMULATIONS

From thousands of performed optimizations **O1-O6** we selected some representable optimizations. The C language programme of more than 14000 lines written by the author was being run on workstations of Hewlett-Packard, Sun, Silicon Graphics, under UNIX operating system. It was being run also on Pentium personal computers under Linux operating system. We used C language UNIX/Linux compilers cc. The time of simulations varied from a couple of hours to several days or even weeks, depending on the discretization parameters. The discretization steps were tested in order to obtain finite solutions to the optimization problems in reasonable time of simulation. They were also studied from the point of view of sensitiveness of optima appearing on them.

6. MATHEMATICAL QUESTIONS

The problem of uniqueness of solutions of optimization problems **O1-O6** is complex. If there are no vehicles allowed in the canyon, then all the six functionals **O1-O6** are constant functions of parameters, the optima exist, and the number of optimal solutions is continuum. The values of functionals are then zero for total travel time and emissions, and are positive for concentrations. These results are analytical. For other cases there are no such analytical results for both the uniqueness and existence known to the author. The numerical solutions

of optimization problems are approximately global within the error induced by discretization of the physical domain Σ and of the domain of control parameters U^{adm}. The method of optimization was a full search.

7. CONCLUSIONS

The proecological traffic control idea and advanced model of the street canyon have been developed. It has been found that the proposed model represents the main features of complex air pollution phenomena.

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