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ON THE APPLICATION OF THE $L_{IC}=(K_{IC}/\sigma_{YS})^2$ CRITERION TO FRACTURE OF CRACKED PIPES

I. M. MIHOVSKY (SOFIA)

The results of certaio experiments on propagation of cracks in internally pressurized pipes are analysed as a known criterion based upon using the quantity L_{IC} as a parameter governing the conditions for crack growth initiation. Formally, L_{IC} is equal to the squared ratio of the material plane strain fracture toughness to its yield stress in pure tension. It is shown that such a criterion is in good agreement with the experiments on crack growth initiation in pipes fabricated from typical ductile structural steels. The criterion is applicable to both through-wall and part-through--wall (surface) cracks.

Introduction

An interesting qualitative approach to establishing the crack growth initiation criterion for ductile material has been recently proposed in [1]. This approach consists in using the material plane strain fractuie to its yield stress ratio. It is shown in the present work that the arguments in [1] lead to real crack growth initiation criterion at least for the class of cylindrical pressure vessels (pipes) fabricated from commercial, ductile structural steels.

1. L_{IC} fracture toughness

When applied to an opening mode crack, the linear elastic fracture mechanics criterion provides for the failure stress value σ and the critical crack length size L an equation of the type

$$K_{IC}=\sigma L^{1/2} f(a_i/L), \tag{1.1}$$

where K_{IC} is the plane strain fracture toughness of the matrrial of the body containing the crack and $f(a_i/L)$ is a dimensionless function depending upon the body geometry through the dimensionless ratios of its typical linear dimensions a_i, $i=1, =1, 2, ..., n$ to the crack length size.

In principle, the linear elasuc fracture mechanics criterion is applicable to ductile materials provided the crack and the body dimensions are sufficiently large, i.e. enormously large when compared with the dimensions of the real structural elements fabricated from commonly used ductile steels. For this reason the linear elastic fracture mechanics criterion is not in fact of great practical importance when crack growth initiation conditions for ductile materials are considered. The latter conclusion is well illustrated by the case of an opening mode crack in an infinite body. In this case Eq (1.1) feduces to the form

$$K_{IC} = \sigma L^{1/2}. \tag{1.2}$$

A simple comparison between the failure stress values (the dashed line in Fig. 1) predicted by Eq. (1.2) and those usually observed in ductile materials (the continuous line in Fig. 1) is the proof of the above conclusion

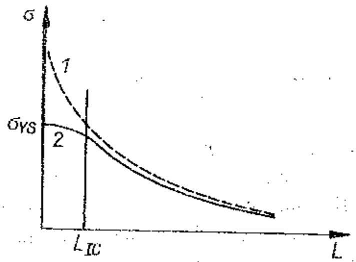

FIG. 1. Failure stress level σ versus critical crack length size L for a ductile material; 1—$K_{IC}^2 = \sigma^2 L$, 2—experiment.

Figure 1 leads at the same time to another important conclusion concerning crack growth initiation and failure in ductile materials. This is the existence of a certain specific length parameter L_{IC} which may be firts regarded as a material constant. For a given ductile material this parameter defines a specific maximum crack length size L_{IC} beyond which the existing crack causes a substantial decrease of the material failure strength, that is, of the stress at which failure of the uncracked material takes place. Since for a ductile material the latter stress is actually the stress at with a developed plastic flow takes place, then the failure strength of such a material is presented by its yield stress σ_{YS}. This implies the following presentation of the L_{IC} parameter (Fig. 1):

$$L_{IC} = (K_{IC}/\sigma_{YS})^2. \tag{1.3}$$

When cracks are present in a ductile material, its failure stress appears to depend strongly on the L/L_{IC} ratio. If such a material contains small cracks ($L/L_{IC} \ll 1$), then its failure is of a ductile type and is governed entirely by the plastic flow mechanism of the material with its yield stress σ_{YS} as a basic plastic flow characteristic. Cracks which are large enough ($L/L_{IC} \gg 1$) lead to brittle fracture; this is governed by the linear-elastic fracture mechanics criterion through the plane strain fracture toughness K_{IC} of the material as its basic characteristic of brittleness. Of utmost importance are obviously the cases in which the crack length size is of the order of magnitude of L_{IC} ($L/L_{IC} \approx 1$). In these cases crack growth initiation takes place when a certain amount of plastic deformation is already present. It is reasonable to suppose that in such cases the crack behaviour will extremely depend upon the interaction between the mentioned mechanisms of rupture with their basic characteristics σ_{YS} and K_{IC}. In accordance with Eq. (1.3) and with the latter considerations, the L_{IC}

parameter may now be regarded as a measure of the interaction of those mechanisms or, equivalently, as a measure of the sensibility of ductile materials to the presence of cracks.

It is well known that the L_{IC} parameter as defined by Eq. (1.3) has already been used in numerous investigations concerning different problems of cracks in inelastic materials such as the near crack tip plastic zone size [2], the geometry of the specimens used for experimental determination of the plane strain fracture toughness K_{IC} [3], the influence of the plate thickness on the fracture surface appearance [4], etc.

The L_{IC} parameter has been considered in [1] as a measure of the material fracture toughness first of all and used as the basis for establishing a crack growth initiation criterion for ductile materials. The main assumption in [1] is that such a criterion may be of the same form as Eq. (1.1) but with the function $f(a_i/L)$ now replaced by a now dimensionless function depending upon the plastic flow characteristics of the material. Thus the proposed form of the criterion is

$$(\sigma/\sigma_{YS}) = (P/L_{IC})\, f(P/L_{IC}, a_i/L_{IC}, G_i), \tag{1.4}$$

where G_i, $i=1, 2, ..., s$ are the mentioned dimensionless plastic flow characteristics such as σ_{US}/σ_{YS} (σ_{US} being the ultimate tensile strength), δ — the elongation at rupture, m — the hardening exponent, etc.

Equation (1.4) accounts for the influence of the temperature and strain rate changes on the crack growth initiation conditions through the quantities L_{IC}, σ_{YS} and G_i. In this respect the following important thing has been pointed out in [1].

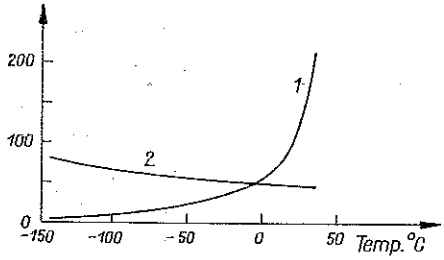

FIG. 2. L_{IC} and σ_{YS} temperature dependence for A533BI steel (σ_{YS}=48 kgfmm^{-1}); 1—L_{IC} (mm) 2—σ_{YS} (kgfmm^{-1}).

Seemingly the temperature and strain rate dependence of the L_{IC} parameter are much stronger than those of σ_{YS} and G_i. Figure 2 for example shows a comparative picture of the temperature dependence of L_{IC} and σ_{TS} for typical ductile steel (A533BI steel). It is seen from the figure that over a range of temperatures from $-150°$ C to 50°C the L_{IC} parameter changes considerably while the order of magnitude of σ_{YS} remains practically unchanged. Similar observations have lead the authors of [1] to the conclusion that the whole temperature and strain rate dependence of the crack growth initiation condition as given by Eq. (1.4) might be considered as entirely governed by the L_{IC} parameter. But this means that, for a certain class of ductile materials with similar dependence of their plastic flow characteristics on the temperature and strain rate changes, the proposed criterion may be reduced to the form

$$(\sigma/\sigma_{YS})^2=(L/L_{IC})\,f\,(L/L_{IC},\ a_i/L_{IC}) \tag{1.5}$$

or, equivalently,

$$\sigma/\sigma_{YS}=F\,(L/L_{IC},\ a_i/L_{IC}). \tag{1.6}$$

It has been supposed in [1] that Eq. (1.6) is thus valid over certain intervals of temperature and strain rate changes.

The following implications of Eq. (1.6) are of great importance. First, it is reasonable to expect that Eq. (1.6) covers the class of commonly used structural steels such as medium-to low-strength steels. Next, if two geometrically-similar cracked bodies are considered with ratios of their typical linear dimensions a_i'/a_i'' and crack length sizes L'/L'' equal to the L_{IC}'/L_{IC}'' ratio, then Eq. (1.6) predicts equal relative failure stress values for these bodies, that is $\sigma'/\sigma_{YS}'=\sigma''/\sigma_{YS}''$. This feature may be extremely important since it makes it possible to determine the failure stress value of a given real structural element by using small models.

The problem of experimental determination of L_{IC} has also been considered in [1]. It has been shown that standard tests on small specimens of Charpy's type might be used for that aim.

2. Equation (1.6) as a crack growth initiation criterion

The most impressive feature of the considered equation seems to be the following. For geometrically-similar cracked bodies fabricated from a given ductile material, this equation states a unique σ/σ_{TS} versus L/L_{IC} relationship exists which, in addition, governs the crack growth initiation conditions over a certain range of temperature changes. In other words, once the function $F(L/L_{IC},\ a_i/L)_{IC}$ in Eq. (1.6) is determined for a given structural element and thus reduced to the function $F(L/L_{IC})$ of only one variable L/L_{IC}, then the same function may be used to predict the failure stress values or the whole class o structural elements which are geometrically similar to the considered one and abricated from the same ductile material.

These observations make Eq. (1.6) especially attractive as a crack growth initiation criterion. However, this equation has not yet been proved to hold as a criterion. On the whole it just summarizes, in quite a natural way, though a number of observations and conclusions concerning crack growth initiation conditions in ductile materials. Thus, from the practical point of view, the most important question of whether that equation may be regarded as a real crack growth initiation criterion is still to be solved. To give a positive ansver to that question means to demenstrate, basing upon Eq. (1.6), the ability of predicting true failure stress values for given structural elements containing given cracks.

In accordance with the basic implications of Eq. (1.6) such an answer may be obtained if it is proved that for a given class of geometrically-similar elements the function analysed above $F(L/L_{IC})$ really exists as a single-valued function of its argument.

The be more explicit suppose that ample experimental data exist on crack growth initiation conditions for ductile materials. Let these data be obtained from tests of a given type and let the tested elements be fabricated from a given ductile material. Let these elements be geometrically similar and the orientation of the crack that each of the elements contains be one and the same. Both the crack length and the test temperature are supposed to vary from one test to another. Finally, let the material yield stress σ_{TS} and L_{IC} fracture toughness be known functions of the temperature at least in the range in which the test temperature varies.

Then the graphical presentation of such data in the terms of σ/σ_{YS} and L/L_{IC} ratios is readily obtained. Equation (1.6) will then be proved to hold as a crack growth initiation criterion if the thus plotted data show a well-pronounced tendency of forming a criteraia smooth σ/σ_{YS} versus L/L_{IC} curve, that is, if the $(\sigma/\sigma_{YS}, L/L_{IC})$ points corresponding to those data form a well-pronounced point representation of a certain smooth curve. That curve will then be an experimentally-obtained graph of the specific (for the considered class of cracked elements) function $F(L/L_{IC})$, the existence of which will therefore be proved. Referring to Fig. 1, it would be reasonable to expect the function $F(L/L_{IC})$ to be a montonically decreasing function.

In accordance with the above considerations it becomes evident now that in order to prove the validity of Eq. (1.6) as a crack growth initiation criterion, one should first possess a sufficient number of experimental data of the described type. Then the next step is appropriate processing of these data.

3. Processing of experimental data

Since the purpose of the present paper was to examine the possibilities of applying Eq. (1.6) as a crack growth initiation criterion to real structures, it was natural first to check if appropriate data exist in the literature instead of performing new experiments.

It is known that a whole range of full-scale experiments was conduced by A. R. Duffy and collaborators some 15 years ago. The experiments were proformed on cylindrical pressure vessels (pipes under internal pressure) containing axial cracks. Both through-wall and part-through-wall (surface) cracks were examined. It seems that the data obtained in the course of these experiments are the most attractive data for our purposes. The author found the most complete presentation of these data in [5]. Partly these data are also given in [6, 7, 8]. All the cylindrical pressure vessels (pipes) tested in the mentioned full-scale experiments were fabricated from typical ductile materials such as medium- to low-strength steels with yield stress levels ranging from 35 to 42 kgfmm^{-2}. The experiments were conducted on pipes of diameters ranging from 150 to 1500 mm and well-thicknesses ranging from 5 to 22 mm. The test temperatures ranged from -140 to 65 °C. The yield stress levels ranged from 35 to 77 kgfmm^{-2}.

It is essential to note that these data do not contain the whole information required for the present purpose. In fact, the work cited above [5] contains only a description of the pipe geometries and the observed failure stress levels and critical crack lengths. For a few cases only the test temperature with the corresponding K_{IC} values are presented in [6]. In general, there is no information directly concerning the plane strain fracture toughness K_{IC} of the materials. Instead, values of the thickness dependent fracture toughness K_C are given in [6]. For these reasons an additional processing of these data has been carried out.

The following procedure has been applied for deriving the necessary information from the data presented in the works cited above.

In [6] the observed failure stress values for through-wall, axial cracks have been used for determining the critical values of the thicknes-dependent fracture toughness K_C. For this reason the following equation has been applied:

$$K_C^2 = L\sigma^2 M^2 (4-k)/2 \cos(\sigma/2\bar{\sigma}).$$

Here $\bar{q}$ is the failure stress level for the uncracked pipe, L is one-half of the crack length, $k=3-4\nu$ for plane strain and $k=(3-\nu)/(1+\nu)$ for plane stress, ν is the Poisson's ratio, M is a curvature correction function introduced by FOLIAS [10] as used in [6] in the form

$$M^2 = 1 + 1.255\,(L^2/Rt) - 0.0135\,(L^4/R^2 t^2)$$

where R and t are the pipe radius and wall-thickness respectively.

Thus the K_C values computed and presented in [6] have been used together with IRWIN's [9] equation

$$(K_C/K_{IC})^2 = 1 + 1.4\,(K_{IC}/\sigma_{YS})^4\, t^{-2}$$

for computing the corresponding K_{IC} values for each of the experiments. For this reason the latter equation has been transformed to the form

$$(K_C/\sigma_{YS})^2 = L_{IC}\,(1 + L_{IC}\, t^{-2}).$$

This equation has been solved graphically for each test, that is for each value of the K_C/σ_{YC} ratio. This is why the obtained L_{IC} values do not satisfy the requirements of high accuracy. Nevertheless, this does not seem to be a great fault. The point is that for each of the steels used in the full-scale experiments the data presented in [6] cover short intervals of temperature changes. This is why the obtained L_{IC} values differ from each other slightly and in most of the cases corresponding mean L_{IC} values have finally been used.

Most of the data presented in [6] concern failure of pipes with an R/t ratio ranging from 40 to 50. For the series of tests conducted on pipes fabricated from API5L $X52$ steel, 21 of the pipes satisfy the condition $R/t=40$. For the pipes fabricated form API5L $X60$ steel. 40 of them have a R/t ratio ranging from 40 to 50. The results obtained for these series by applying the procedure described above are presented in Table 1 and Table 2, respectively, and plotted in Fig. 3.

Table 1

Steel API5L X52, L_{IC}=31.6 mm, R/t=40.

σ/σ_{YS}	$2L/L_{IC}$	σ/σ_{YS}	$2L/L_{IC}$	σ/σ_{YS}	$2L/L_{IC}$	σ/σ_{YS}	$2L/L_{IC}$
0.45	7.0	1.20	0.8	0.55	5.6	0.45	5.0
0.45	7.0	1.20	0.8	0.52	5.6	0.78	2.4
0.43	7.0	0.92	2.7	0.60	5.6	0.25	11.2
0.43	7.0	0.94	2.7	0.60	5.6	0.63	5.0
—	—	—	—	—	—	0.76	3.2

Table 2

Steel API5L X60, L_{IC}=28.2 mm, R/t=40—50.

σ/σ_{YS}	$2L/L_{IC}$	σ/σ_{YS}	$2L/L_{IC}$	σ/σ_{YS}	$2L/L_{IC}$	σ/σ_{YS}	$2L/L_{IC}$
0.48	4.8*	0.40	7.8′	0.75	4.8′	0.98	2.3
0.61	5.7*	0.50	5.3′	0.72	4.8′	0.60	4.6 (50)
0.60	5.7*	0.40	9.9′	0.75	4.8′	0.63	4.9 (50)
0.56	5.7*	0.71	4.7′	0.84	4.8′	0.58	4.9 (50)
0.55	5.7*	0.70	4.7′	0.78	4.5′	0.37	11.7 (50)
0.27	13.5*	0.70	4.7′	0.85	4.8′	0.62	5.7 (47)
0.24	17.9*	0.66	4.7′	0.75	4.8′	0.70	4.8 (47)
0.76	4.8*	0.65	4.7′	0.84	4.8′	0.23	17.6 (42)
0.77	4.9*	0.68	4.7′	0.80	4.8′	0.48	7.1 (42)
0.32	9.5*	0.65	4.7′	0.70	4.8′	0.60	4.6 (42)

*) R/t=40, ′) R/t=54, (—)) R/t=(—)

Table 3

Steel API5L X52, R/t=40.

σ/σ_{YS}	$2L/L_{IC}$	Temp°C	L_{IC}mm	σ/σ_{YS}	$2L/L_{IC}$	Temp°C	L_{IC}mm
0.43	7.6	24	29	0.31	9.6	−27	23
0.43	7.4	− 2	29	0.30	10.3	−60	21.5
0.45	7.4	−19	30	0.08	26.1	−96	8.5
0.45	7.5	−20	29.5	—	—	—	—

Table 4

Steel API5L X52, L_{IC}=31.6 mm, R/t=40.

σ/σ_{YS}	$2L/L_{IC}$	d/t	σ/σ_{YS}	$2L/L_{IC}$	d/t	σ/σ_{YS}	$2L/L_{IC}$	d/t
0.85	12.1	0.4	0.58	12.0	0.6	0.23	12.1	0.8
0.89	7.2	0.4	1.05	2.7	0.6	0.84	2.9	0.8
1.08	2.7	0.4	1.07	2.7	0.6	—	—	—
0.67	7.1	0.6	0.38	7.3	0.8	—	—	—

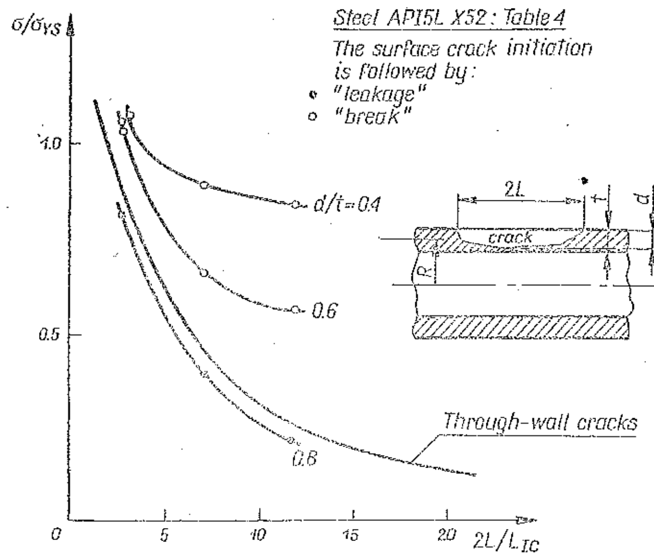

Fig. 3. σ/σ_{YS} versus $2L/L_{IC}$ curve for through-wall cracks.

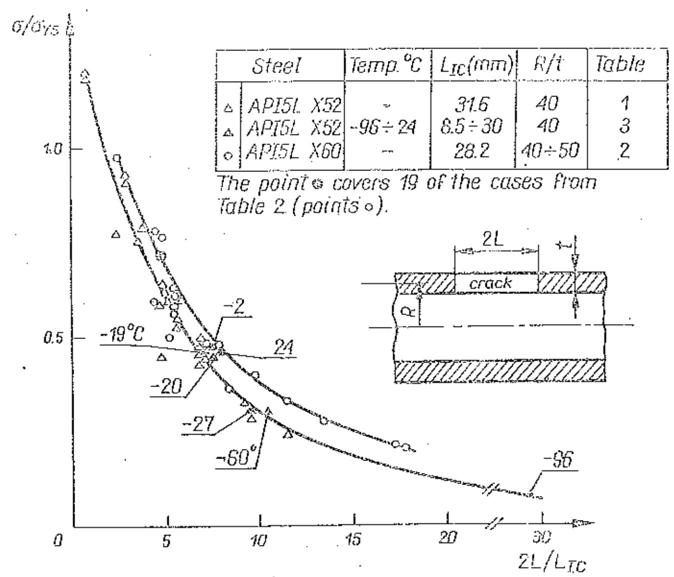

Fig. 4. σ/σ_{YS} versus $2L/L_{IC}$ curve for surface cracks with different d/dt ratios.

Of special interest are the results given in Table 3 where the L_{IC} values are presented together with the corresponding test temperatures. These results are also plotted in Fig. 3.

With the obtained values of L_{IC} the data in [6] concerning the behaviour of part-through-wall cracks have been processed in a similar way. In this case the condition of geometrical similarity requires that only pipes of equal R/t and d/t ratios be considered, d being the crack depth. The results obtained for surface cracks are given in Table 4 and Fig. 4.

4. Conclusions

Figure 3 shows first that the function $F(L/L_{IC})$ in Eq. (1.6) really exists. Thus the latter equation may be regarded as a real crack growth initiation criterion for ductile materials and called a L_{IC} criterion. Figure 3 reveals at the same time the most important feature of that criterion: the fact that the L_{IC} criterion in a simple way governs a great variety of concrete situations. The curve for API5L X52 steel series in that figure governs the crack growth initiation conditions for pipes of diameters ranging from 350 to 450 mm and wall-thickness from 7 to 13 mm, crack lengths ranging from 15 to 500 mm and temperatures ranging at least from −96 to 24°C.

Figure 4 confirms the applicability of the L_{IC} criterion to surface cracks as well. For three different geometries of pipes, that is for three different values of the d/t ratio, the corresponding functions $F(L/L_{IC})$ are determined experimentally and compared with the function governing the through-wall cracks growth initiation conditions. It is known that the growth of a surface crack once initiated may be followed by either "leakage" or a "break" [11]. To prevent the unstable (catastrophic) crack propagation means to ensure, for a given depth of the crack, such a length that the crack growth initiation is followed by "leakage". It is interesting to note that, in general, the situations in which "break" has been observed in the full-scale experiments are presented in Fig. 4 by curves lying above the curve for the through-wall cracks. The curve corresponding to $d/t=0.8$ is close to the through-wall cracks curve but lies bellow it and corresponds to situations in which "leakage" has been mainly observed. This observation suggests that it would be advisable to check if such situations will take place for larger series of tests on propagation of surface cracks. If this is the case, then the L_{IC} criterion may successfully be used for a simple prediction of surface cracks propagation.

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STRESZCZENIE

O STOSOWANIU KRYTERIUM ZNISZCZENIA $L_{IC}=(K_{IC}/\sigma_{YS})^2$ DO RURZE SZCZELINAMI

Wyniki pewnych doświadczeń nad propagacją szczelin w rurach poddanych ciśnieniu wewnętrznemu przeanalizowano w świetle znanego kryterium opartego na stosowaniu wielkości L_{IC} jako parametru określającego warunki wzrostu szczeliny. Parametr ten równy jest kwadratowi stosunku odporności materiału na pękanie w płaskim stanie odkształcenia do jego granicy plastyczności przy czystym rozciąganiu. Wykazano, że lkryterium takie sprawdza się dobrze w doświadczeniach prowadzonych na rurach wykonanych z typowych ciągliwych stali konstrukcyjnych. Kryterium stosować można w przypadku ścianek pękniętych na wskroś lub szczelin powierzchniowych.

Резюме

О ПРИМЕНЕНИИ КРИТЕРИЯ РАЗРУШЕНИЯ $L_{IC}=(K_{IC}/\sigma_{YS})^2$ ДЛЯ ТРУБ СО ЩЕЛЯМИ

Результаты некоторых экспериментов по распространению щелей в трубах, подвергнутых внутреннему давлению, проанализированы в свете известного критерия, опирающегося на применению величины L_{IC}, как параметра определяющего условия роста щели. Этот параметр равен квадрату отношения стойкости материала на растрескивание в плоском деформационном состоянии к его пределу пластичности при чистом растяжении. Показано, что такой критерий хорошо проверяется в экспериментах, проведенных на трубах изготовленных из типичных тягучих конструкционных сталей. Критерий можно применять в случае стенок разрушенных насквозь или поверхностных щелей.

SOFIA UNIVERSITY, SOFIA, BULGARIA

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