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ON A CERTAIN CLASS OF STANDARD STRESS FIELDS FOR PLASTIC DESIGN

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The paper presents a certain type of statically admissible stress fields that can be useful in plastic design of structural elements of complex shape. It is shown that these stress fields, constructed with the use of the method of characteristics, give better (more economical) estimates of designed shapes than the fields obtained by means of the piecewise-homogeneous stress field technique. Plastic design of a transversely loaded beam is presented as an example of practical application.

1. Introduction

A number of papers have been devoted to plastic design of structural elements of complex shape, such as machine elements or connections of steel structures. These papers contain numerous theoretical solutions as well as examples of experimental verification. An extensive description of this method of strength design and a review of the results obtained is presented in the monograph by Szczepiński and Szlagowski [3].

Plastic design (or limit capacity design) is based upon extremum principles of the mechanics of plastic flow. According to these principles, upper bound on the load bearing capacity of a structural element may be found from any kinematically admissible collapse mechanism. Lower (safe) bound on the load bearing capacity may be found from any statically admissible stress field, i.e. such field that satisfies the equilibrium equation and stress boundary conditions and in any point of which the yield condition is not violated.

When the shape of a structural element is not prescribed in advance but is to be designed, the outer contour of the statically admissible stress field gives us safe estimate of this shape.

Two general methods of construction of stress fields for plastic design are applied. The first one, described in detail in [3], consists in creating piecewise-homogeneous stress fields, i.e. fields composed of a certain number of homogeneous sub-fields separated by the lines of stress discontinuity.

It turns out however, that in many cases similar but more economical solutions may be found by means of the method of characteristics (slip-lines), well-known from the mechanics of plastic flow.

This paper describes a certain type of statically admissible stress fields of this kind. These fields may be used as standard elements in larger stress systems, replacing analogous piecewise-homogeneous solutions. The results are presented in a form suitable for practical use. Possibilities of application are illustrated by an example.

2. Formulation of the problem

Consider a plane stress field, generated by external load distributed along the boundary $ABB'A'$ (Fig. 1). The loaded boundary consists of three sections: AB, BB', and $B'A'$, whose lengths are respectively a, $2b$ and a. The outer sections form angles ε with the central, horizontal one $(0 \leq \varepsilon < \pi/2)$.

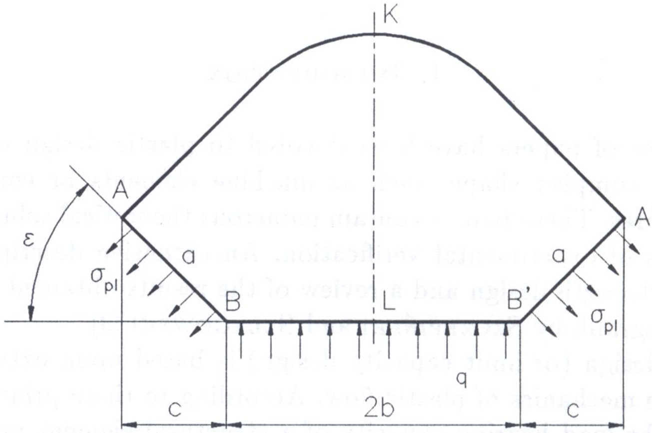

Fig. 1.

Suppose, that the load is at every point normal to the boundary, and that along the sections AB and $A'B'$ it is equal to the yield stress $\sigma_{\rm pl}$, while along the BB' it equals $q < 0$. Tractions q must satisfy the equilibrium condition

$$q = -\sigma_{\rm pl}c/b = -\sigma_{\rm pl}a(\cos\varepsilon)/b. \tag{2.1}$$

The boundary AKA' is stress-free and is *a priori* unknown. The problem consists in constructing statically admissible stress field compatible with the above boundary conditions, external contour of which will determine the boundary AKA'. We are looking for the most economical solution, i.e. such estimate, that requires a possibly small volume of the material.

It is assumed, that the stress field is symmetrical with respect to the axis KL. It is also assumed, that the element under consideration is of constant thickness, that the stresses perpendicular to the plane of drawing vanish and that the material obeys the Tresca yield criterion. Thus, the principal stresses in the plane of drawing must satisfy the inequalities

$$(2.2) \qquad |\sigma_1| \le \sigma_{\text{pl}}, \qquad |\sigma_2| \le \sigma_{\text{pl}}, \qquad |\sigma_1 - \sigma_2| \le \sigma_{\text{pl}}.$$

3. Piecewise-homogeneous solutions

Elementary solution of the above defined problem is shown in Fig. 2. The stress field consists of three homogeneous sub-fields ABC, $BB'C$ and

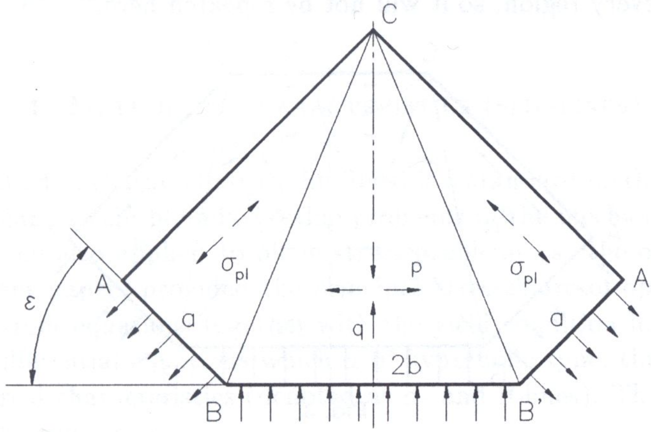

FIG. 2.

$A'B'C$. The material in the regions ABC and $A'B'C$ undergoes uniaxial tension equal to the yield stress σ_{pl}. BC and $B'C'$ are stress discontinuity lines. In the region $BB'C$ there is a homogeneous biaxial state of stress. The stress in vertical direction q is given by the formula (2.1) while the stress p (horizontal) is equal to

$$(3.1) \qquad p = \frac{(c/b)\,\text{tg}^2\varepsilon}{1 + (c/b)(1 + \text{tg}^2\varepsilon)}\sigma_{\text{pl}}.$$

The stress field remains statically admissible for the values of the characteristic parameter c/b within the range $0 < c/b \le \cos\varepsilon$. In the limit case $c/b = \cos\varepsilon$, we get the standard system of discontinuity lines, described in [1–3]. There is $\kappa = \lambda = \varepsilon/2$, $a = b$ and

$$p = \frac{\sin^2\varepsilon}{1 + \cos\varepsilon}\sigma_{\text{pl}}.$$

Since in this case $p - q = \sigma_{\text{pl}}$, the material in the region $BB'C$ is in the plastic state.

The above stress field has a simple structure and can be easily constructed for a wide range of characteristic parameters ε and b/c. It gives, however, rather crude estimate of the optimum shape, especially for small values of the angle ε.

Better estimates may be obtained by means of more complicated stress fields shown in Figs. 3 and 4. The first one consists of six homogeneous sub-fields and its stress-free contour includes two corners C and C'. The area of the field is thus smaller than in the previous case. The construction of this field (denoted as elementary stress field of type A) is described in detail in [3]. This description includes formulae for calculation of the state of stress in every region, so it will not be repeated here.

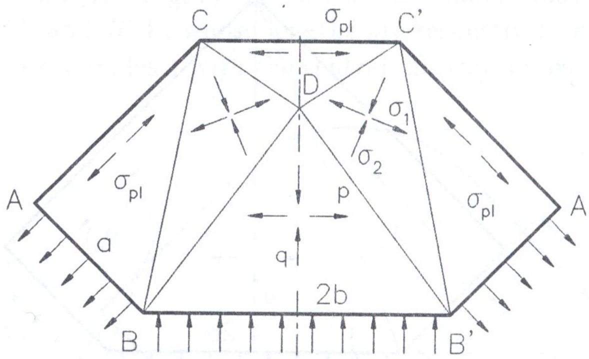

FIG. 3.

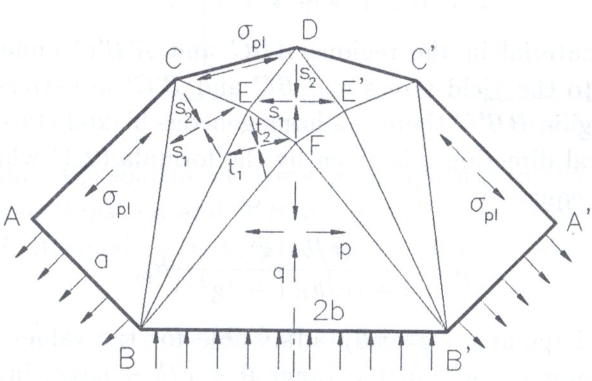

FIG. 4.

The monograph [3] contains also a detailed description of the second field (Fig. 4), denoted as the elementary stress field of type B. This field is composed of ten homogeneous sub-fields, and its free boundary includes three corners. And again, the area of this field is smaller than that of the field of type A.

So, increasing the number of homogeneous regions within the stress field and the number of corners at the free boundary, may result in a more economical solution. This observation led to the supposition formulated already in monograph [1], that the best solution of this problem (for the case $\varepsilon = 0$ and $c/b = 1$) can be found as a limit of a certain sequence of solutions. Each consecutive solution in this sequence includes greater amount of homogeneous regions, while the differences between the stress tensors in most of the neighbouring regions become smaller.

This limit can be found by direct application of the slip-line method. The solution for a more general case $0 \leq \varepsilon < \pi/2$ and varying values of the c/b ratio will be presented later on.

4. Method of characteristics (slip-lines)

The method of characteristics (slip-lines) is a standard method of solving of various plane strain boundary-value problems in the mechanics of plastic flow. It can be also applied to plane stress problems (as the one discussed in the present paper) provided the principal stresses are of opposite signs. The equilibrium equations together with the yield condition form a system of partial differential equations which is of hyperbolic type, thus it has two families of real characteristics (denoted as α- and β-lines). They are determined by the equations

$$(4.1) \qquad \frac{dy}{dx} = \operatorname{tg}\left(\varphi + \frac{\pi}{4}\right), \qquad \chi + \varphi = \text{const} \qquad (\alpha\text{-family}),$$

$$(4.2) \qquad \frac{dy}{dx} = \operatorname{tg}\left(\varphi - \frac{\pi}{4}\right), \qquad \chi - \varphi = \text{const} \qquad (\beta\text{-family}),$$

where the function

$$(4.3) \qquad \chi = \frac{1}{2}(\sigma_1 + \sigma_2)/\sigma_{\text{pl}}$$

($\sigma_{1,2}$ – the principal stresses), and φ is the angle between the x axis and the direction of σ_1 (larger principal stress). Since for the Tresca yield condition there is $\sigma_1 - \sigma_2 = \sigma_{\text{pl}}$, thus we have

$$(4.4) \qquad \sigma_1 = \left(\chi + \frac{1}{2}\right)\sigma_{\text{pl}}, \qquad \sigma_2 = \left(\chi - \frac{1}{2}\right)\sigma_{\text{pl}}.$$

It follows from (4.1) and (4.2), that characteristics of both families form an orthogonal net and are inclined at angles $\pi/4$ to the principal directions, so they coincide with the lines of maximum shear stress (slip-lines). The method of solution consists in numerical integration of the equations of characteristics. As a result, one obtains the coordinates of all the nodes of the net and the corresponding values of the functions χ and φ describing the state of stress at each node. The method of characteristics is described e.g. in [3] or in any other monograph or textbook concerning mechanics of plastic flow or its applications and it will not be presented here in more detail.

5. Slip-line solution

The solution to the problem under consideration, obtained by means of the method of characteristics is presented in Fig. 5. There are two slightly different versions of this stress field shown respectively on the left and on the right-hand side of the axis of symmetry.

The external loads distributed along $ABB'A$ are the same as for the stress fields presented above. The boundary AB is loaded by uniaxial uniform tensions of magnitude σ_{pl}. The same state of stress exists within the whole region ABC bounded by the slip-line BC which belongs to the α-family. At the point B we have discontinuity of the stress boundary conditions. Assume that this point is a center of a fan of slip-lines of the α-family with the angular span equal γ. The α-lines within the fan are straight, so, along these lines $\varphi = \text{const}$ and, according to the formula $(4.1)_2$, $\chi = \text{const}$ i.e. the state of stress does not change. It does however change along the β-lines, which are the family of circular arcs with the center at B. The curve CD belongs also to this family. The state of stress at the point C is the same as within the triangle ABC, i.e. $\sigma_{1C} = \sigma_{\text{pl}}$, $\sigma_{2C} = 0$, $\varphi_C = \pi/2 - \varepsilon$, and, according to the formula (4.3), $\chi_C = 0.5$. At the point D we have

$$\varphi_D = \pi/2 - \varepsilon - \gamma. \tag{5.1}$$

It follows from Eq. $(4.2)_2$, that

$$\chi_D = \chi_C - \varphi_C + \varphi_D = 0.5 - \gamma. \tag{5.2}$$

Thus we have

$$\sigma_{1D} = (1-\gamma)\sigma_{\text{pl}}, \qquad \sigma_{2D} = -\gamma\sigma_{\text{pl}}. \tag{5.3}$$

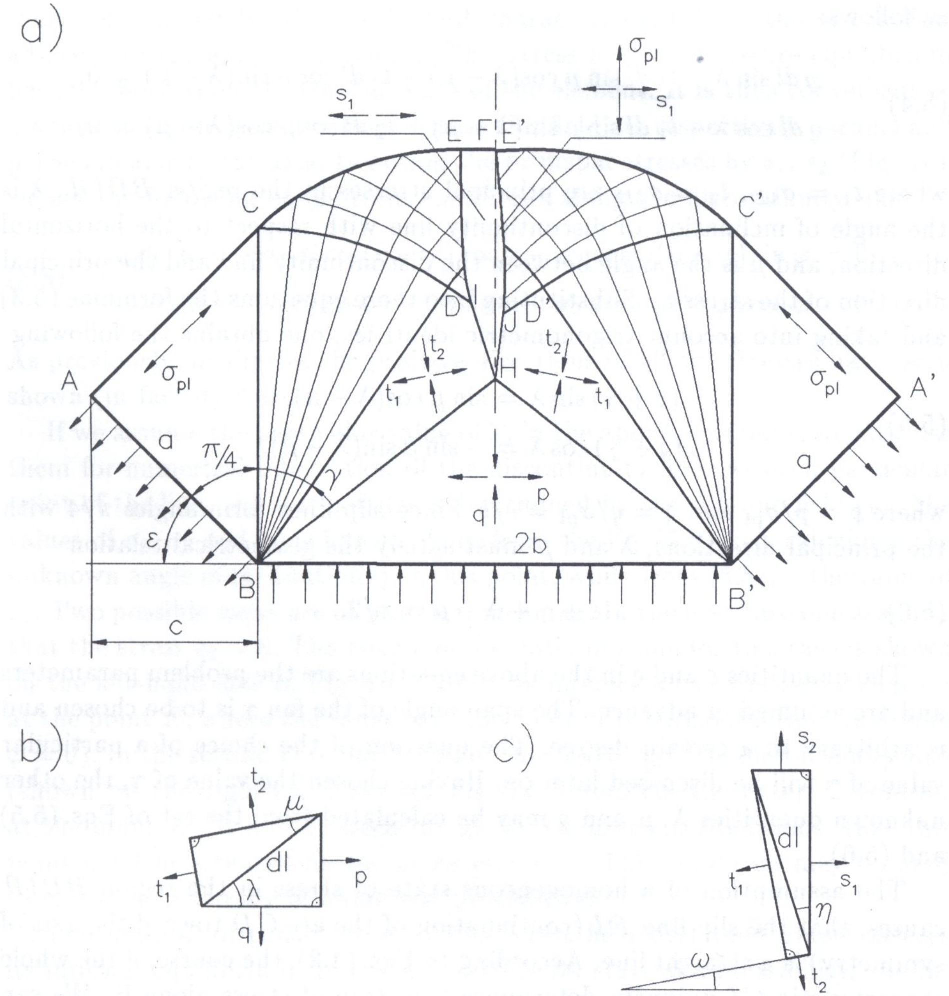

FIG. 5.

The same state of stress exists at every point of the line BD and is assumed to exist within the whole region $BDIH$.

We also assume existence of the homogeneous state of stress in the triangle BHB' adjacent to the boundary BB'. In order to assure compatibility with the boundary conditions on BB', the principal directions in this region must be respectively perpendicular and parallel to the boundary, and the principal stress in the perpendicular direction must be equal q. BH is the discontinuity line. Figure 5 b shows an element of this line (the arrows point at the positive directions of all the stresses). The equilibrium equations are as follows:

$$(5.4)\qquad \begin{aligned} &p\,dl \sin\lambda - t_1\,dl \sin\mu\cos(\lambda-\mu) - t_2\,dl\cos\mu\sin(\lambda-\mu) = 0,\\ &-q\,dl\cos\lambda - t_1\,dl\sin\mu\sin(\lambda-\mu) - t_2\,dl\cos\mu\cos(\lambda-\mu) = 0, \end{aligned}$$

where $t_1 = \sigma_{1D}$, $t_2 = \sigma_{2D}$ are principal stresses in the region $BDIH$, λ is the angle of inclination of discontinuity line with respect to the horizontal direction, and μ is the angle between the discontinuity line and the principal direction of the stress t_1. Substituting into these equations the formulae (5.3) and taking into account trigonometric identities, one obtains the following

$$(5.5)\qquad \begin{aligned} (\mathfrak{p}+\gamma)\sin\lambda &= \sin\mu\cos(\lambda-\mu),\\ (\mathfrak{q}+\gamma)\cos\lambda &= -\sin\mu\sin(\lambda-\mu), \end{aligned}$$

where $\mathfrak{p} = p/\sigma_{\rm pl}$ and $\mathfrak{q} = q/\sigma_{\rm pl} = c/b$. Since slip-lines form angles $\pi/4$ with the principal directions, λ and μ must satisfy the geometrical relation

$$(5.6)\qquad \varepsilon + \gamma + \lambda - \mu = \pi/2.$$

The quantities ε and $\mathfrak{q}$ in the above equations are the problem parameters and are assumed in advance. The span angle of the fan γ is to be chosen and is arbitrary to a certain degree. The question of the choice of a particular value of γ will be discussed later on. Having chosen the value of γ, the other unknown quantities λ, μ and $\mathfrak{p}$ may be calculated from the set of Eqs. (5.5) and (5.6).

The assumption of a homogeneous state of stress in the region $BDIH$ causes, that the slip-line DI (continuation of the arc CD toward the axis of symmetry) is a straight line. According to Eqs. (4.2), the course of the whole characteristic CI uniquely determines the state of stress along it. We can then solve the so-called inverse Cauchy problem based upon the known state of stress along CI and the condition that the boundary CE is stress-free. The relations that determine this boundary are

$$(5.7)\qquad \frac{dy}{dx} = \operatorname{tg}\varphi, \qquad \chi = 0.5.$$

The numerical procedure of solving the inverse Cauchy problem in such case is described in [3]. The solution determines the unknown course of the stress-free boundary CE and the stress field within the whole region $CDIE$.

The discontinuity line BH intersects the axis of symmetry at the point H. This point is assumed as the beginning of another discontinuity line, separating the stress field being a solution of the inverse Cauchy problem (and constructed by the method of characteristics) from the stress field adjacent to the axis of symmetry. This stress field must assure equilibrium between both the symmetrical parts of the element. It is thus convenient to assume that principal directions within this field are respectively parallel and perpendicular to the axis. Denoting the principal stresses by s_1, s_2 (Fig. 5 c), we get the following equilibrium conditions along the discontinuity line

$$\begin{aligned} &s_1\, dl \cos\eta - \sigma_1\, dl \cos(\eta-\varphi)\cos\varphi + \sigma_2\, dl \sin(\eta-\varphi)\sin\varphi = 0, \\ &s_2\, dl \sin\eta - \sigma_1\, dl \cos(\eta-\varphi)\sin\varphi - \sigma_2\, dl \sin(\eta-\varphi)\cos\varphi = 0. \end{aligned} \tag{5.8}$$

As previously, in Fig. 5 c the positive directions of all the stresses have been shown (in fact $\sigma_2 \le 0$).

If we assume the particular value of s_2 in the above equations, we may use them for numerical integration of the discontinuity line. At each particular point of the line, the state of stress determined by the slip-line field (i.e. the values of σ_1, σ_2 and φ) is known. Thus from the Eq. $(5.8)_2$ one calculates the unknown angle of inclination η at this point, while from $(5.8)_1$ – the value of s_1. Two possible cases are of practical meaning. In the first case one assumes that the stress $s_2 = 0$. The course of discontinuity line for this case is shown on the left-hand side of Fig. 5 a. The curve intersects the stress-free contour at the point E, where the tangent to the contour is horizontal (and therefore $\varphi = 0$). In the second case one assumes that $s_2 = \sigma_{\text{pl}}$. The discontinuity line (shown on the right-hand side of Fig. 5 a) intersects the external contour at the point E'. It follows from the global equilibrium conditions, that this point must be situated on the symmetry axis. This condition may be used to check the accuracy of numerical procedures.

The method of construction of the stress field described above assures its internal equilibrium. In order to assure its statical admissibility, the inequalities (2.2) must be satisfied. This imposes certain limitations on the value of the angle γ. Firstly, it follows from the Eq. $(5.3)_2$, that

$$\gamma \le 1. \tag{5.9}$$

Then the angle γ must be chosen in such a way, that the value of $\mathfrak{p}$ calculated from the Eqs. (5.5) and (5.6) will satisfy the inequalities

$$|\mathfrak{p}| \le 1, \qquad |\mathfrak{p} - \mathfrak{q}| \le 1. \tag{5.10}$$

At the end, the stress s_1 must not violate the yield condition. Thus, for the case shown on the left-hand side of the axis of symmetry (Fig. 5 a), it must be

$$|s_1| \le \sigma_{\text{pl}}\,, \tag{5.11}$$

while for the case on the right-hand side of it, the condition

$$\sigma_{\mathrm{pl}} \geq \sigma_{\mathrm{pl}} - s_1 \geq 0 \tag{5.11'}$$

must be satisfied.

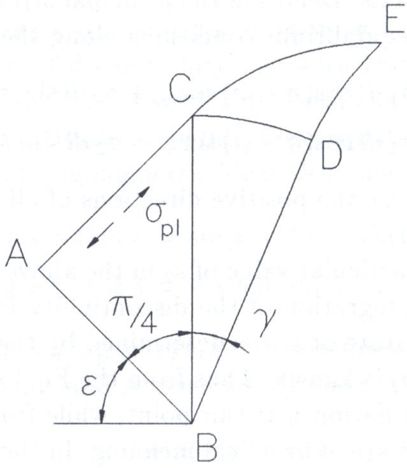

FIG. 6.

The above conditions should be accompanied by geometrical ones. The first condition consists in the requirement, that the angle between the tangent to the stress-free contour and the external direction of the axis of symmetry is equal to $\pi/2$ or less, as it is in Fig. 5. In other words, the value of the angle φ at the point of intersection of the external slip-line BDE of the fan with the stress-free contour (Fig. 6)

$$\varphi_E \leq 0. \tag{5.12}$$

According to Eq. $(4.1)_2$, there is

$$\chi_D + \varphi_D = \chi_E + \varphi_E\,, \tag{5.13}$$

and from (5.7) we have

$$\chi_E = 0.5. \tag{5.14}$$

It follows from the equalities (5.1), (5.2), (5.13) and (5.14), that the condition (5.12) is equivalent to

$$\pi/4 - \varepsilon/2 \leq \gamma. \tag{5.15}$$

Although it may be possible to build a statically admissible stress field which does not satisfy the above inequality, it would have different structure than the stress field shown in Fig. 5. Such stress field would be of minor practical importance.

The second geometrical condition consists in the requirement that the angle of inclination of the direction of the principal stress σ_1 at the point D (Fig. 5) and within the whole area $BDIH$ should be positive, i.e.

$$\varphi_D \geq 0. \tag{5.16}$$

It follows from the equality (5.1), that this is equivalent to the condition

$$\gamma \leq \pi/2 - \varepsilon. \tag{5.17}$$

When the value of γ comes close to $\pi/2-\varepsilon$, then the point H moves towards the loaded boundary BB'. At the limiting value of γ the boundary and the discontinuity line BH coincide. It is not possible to build a stress field of the structure described above for larger values of γ.

Thus, for a particular problem defined by the angle of inclination of the loaded external boundaries ε and the c/b ratio (denoted by $\mathfrak{q}$), the angle γ must satisfy the inequalities (5.9), (5.15) and (5.17). The choice of γ within these limits is free provided the conditions (5.10) and (5.11) are satisfied. It must be emphasized, that this choice does not affect the shape of the external contour of the field. The problem presented in Fig. 5 was defined by the values $\varepsilon = \pi/4$ and $\mathfrak{q} = c/b = 0.707$, and the stress field has been constructed for $\gamma = 0.600$.

6. Approximation for practical applications

Similar solutions may be found for various combinations of characteristic parameters ε and $\mathfrak{q}$. In paper [5] the solution for $\varepsilon = 0$ and $\mathfrak{q} = 0.667$ was presented, and in [6] another one for $\varepsilon = 0$ and $\mathfrak{q} = 1$. Stress-free contours for all these solutions are, according to the prior assumption, symmetrical with respect to vertical axis. Each half of such contour consists of a straight segment and a curve, which either ends with another segment perpendicular to the axis or intersects the axis at an angle close to $\pi/2$. It would be troublesome and unnecessary to calculate the course of this curve and then to reproduce it in a real construction for every possible set of parameters ε and $\mathfrak{q}$. Instead of this, the contour may be, with a very good accuracy, approximated by segments and circular arcs. Their dimensions are tabulated and the choice of their appropriate values requires very few calculations (if any).

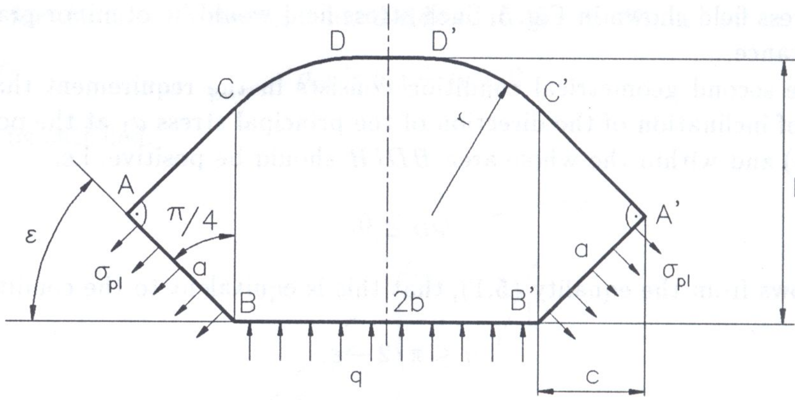

FIG. 7.

Consider the contour shown in Fig. 7. The loaded segment AB has the length a. It is convenient to use this dimension as a reference which determines the magnitude of the whole field. The other dimensions are proportional to a; $c = a/\cos\varepsilon$ and $b = c/\mathfrak{q} = a/(\mathfrak{q}\cos\varepsilon)$. Thus, a may be taken as equal to unity.

The stress-free contour consists of three straight segments AB, DD' and $A'C'$, connected by circular arcs CD and $C'D'$. The segment AC is perpendicular to AB and has the same length a. Thus, the position of the point C in relation to B is uniquely determined by the value of the angle ε. The dimension h is defined as the elevation of the highest point of the slip-line field over the base BB'. It is easy to see from Fig. 5 that for γ satisfying the condition (5.15), h also depends only on ε. Line DD' is parallel to BB'. The circular arc approximating the curve CE of Fig. 5 is defined as tangent to AC at the point C and to the line DD' determined by the distance h. These two conditions uniquely determine the arc and its radius r. The end of the arc determines the position of the point D. The whole arc lies outside the curve CE of Fig. 5, so the distance between the point D (Fig. 7) and the axis of symmetry is slightly greater than that between the point E (Fig. 5) and the axis. The approximation in Fig. 7 is thus safe.

The course of the contour ACD in relation to B is determined uniquely by the parameter ε. Only the length of DD' depends on $\mathfrak{q}$, and may be easily calculated from the other geometrical quantities. Table 1 presents the calculated values of h and r for the most commonly used values of the angle ε. The table also shows the minimum and maximum values of the second characteristic parameter $\mathfrak{q}$ within which the stress field of Fig. 5 is possible to construct. They result from the conditions discussed at the end of the preceding section.

Table 1.

ε	h/a	r/a	$\mathfrak{q}_{\min}$	$\mathfrak{q}_{\max}$
0	2.276	1.276	0.635	1.000
0.262 (15°)	2.134	1.227	0.523	1.000
0.524 (30°)	1.955	1.179	0.426	1.000
0.785 (45°)	1.746	1.132	0.332	0.785
1.047 (60°)	1.512	1.088	0.232	0.524
1.309 (75°)	1.260	1.045	0.123	0.262

For the particular set of parameters ε and $\mathfrak{q}$, one has to check first if $\mathfrak{q}$ does not exceed the above specified limits, and then, taking the values of h and r from the table, one can determine with a good accuracy the course of the stress-free contour. The approximation similar to the one just described, was given in [1], for the particular case $\varepsilon = 0$, $\mathfrak{q} = 0.667$. The recommended solution was equivalent to $h/a = 2.5$ and $r/a = 1.5$, so it was less economical than the present one.

The stress field described here may be used directly e.g. to design the shape of a head of a tension member (as in [5] or [6]), or as an element of a larger stress system – constructed partly or as a whole by means of the piecewise-homogeneous stress field technique. In this case the present field simply replaces the standard system of discontinuity lines of Fig. 2 or the elementary stress fields of type A or B. In analogy to this notation, we shall denote the present field as a standard stress field of type T. The example of its application within the larger stress system will be shown in the next section.

7. Example: design of a transversely loaded beam

Consider a structural element shown in Fig. 8. It is a short beam loaded transversely in its center and supported at the ends. The external tractions of the magnitude σ_{pl} are uniformly distributed along the segment GG' of the upper edge of the beam. The total load is equal to $F = \sigma_{\text{pl}} 2ag$ ($2a$ – length of the segment, g – thickness of the beam). This load is in equilibrium with the bearing tractions of the same magnitude σ_{pl} distributed along AB and $A'B'$. If the assumed distance d between the loaded segment GG' and the plane of supports AA' cannot be too large, some parts of the beam must be situated below the plane AA'. Figure 8 shows possible solution to such a problem.

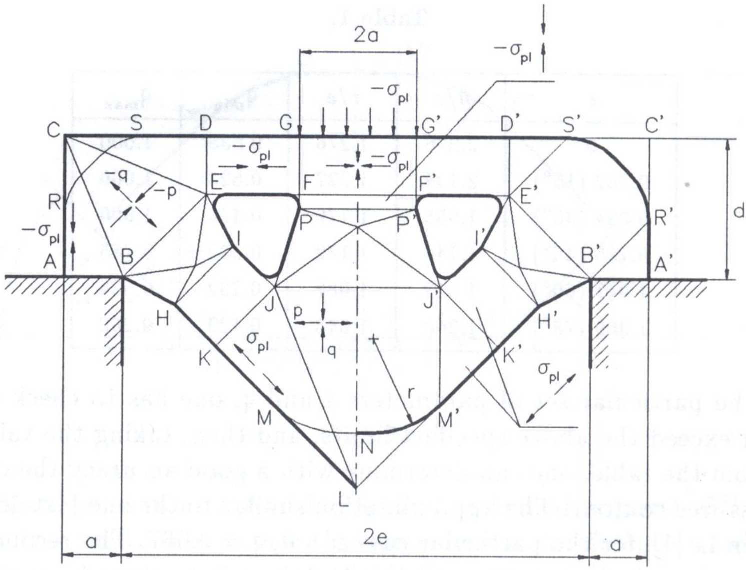

FIG. 8.

The piecewise-homogeneous stress field is presented in some detail on the left-hand side of the axis of symmetry. The whole stress field includes three subfields of the type shown in Fig. 2. Two of them, $BACDE$ and $B'A'C'D'E'$ (not shown here directly), have the stresses of signs opposite to those of Fig. 2, while the third one, $JKLK'J'$, has the same signs. All three have been drawn for $\varepsilon = \pi/4$ and $\mathfrak{q} = c/b = 0.707$, but $JKLK'J'$ is larger than the other two; its characteristic dimension is $a\sqrt{2}$ instead of a. Besides, the stress field includes three other subfields $BEIH$, $B'E'I'H'$ and $JJ'P'P$ of the type originally proposed by WINZER and CARRIER [4]. These subfields change the magnitude of uniaxial stress from p to σ_{pl}. Their structure is described also in [3]; they are denoted there as elementary stress fields of type D. The connection of subfields of these two types for the above specified values of ε and $\mathfrak{q}$ has been presented in [7]. The regions $HIJK$ and $H'I'J'K'$ undergo uniaxial tension equal to the yield stress σ_{pl}, while $DEFG$, $D'E'F'G'$ and $FF'P'P$ are uniaxially compressed also to the yield point. The region $GG'F'F$ undergoes biaxial compression of the same magnitude and the regions $EFPJI$ and $E'F'P'J'I'$ are stress-free, so no material is needed there.

The whole stress field is in equilibrium and the yield condition is nowhere violated. Thus, the contour of this field, drawn by the thick line on the left-hand side of the axis of symmetry constitutes the safe design of the beam. This solution may be, however, improved by replacing the subfields $BACDE$, $B'A'C'D'E'$ and $JKLK'J'$ with the fields of type T presented in this paper. The corrected contour is shown on the right-hand side of the axis of symmetry. One can easily observe, that this solution is more economical, has smaller cross-section in the center and a rounded shape. In fact, most of the piecewise-homogeneous solutions may be improved in this manner.

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