ENGINEERING TRANSACTIONS • Engng. Trans. • 44, 3–4, 349–360, 1996
Polish Academy of Sciences • Institute of Fundamental Technological Research

A NOTE ON THE CRACK PROBLEM FOR A NONHOMOGENEOUS PLANE

P. K. CHAUDHURI and S. RAY (CALCUTTA)

A plane elasticity problem in a nonhomogeneous medium containing a crack has been considered. It is assumed that the medium has a constant Poisson's ratio while the Young's modulus varies exponentially both along and perpendicular to the length of the crack. The problem is solved by derivation of an integral equation and the effects of nonhomogeneity have been shown in tables and graphs.

1. Introduction

Crack problems in nonhomogeneous media have been studied for the last few decades due to their practical importance in predicting failure of various elastic materials. In a medium containing a crack, stresses usually exhibit singular behaviour at the crack tip. But the usual square root singularity for a homogeneous isotropic medium is not always observed in nonhomogeneous medium. Although a nonhomogeneous medium with continuous and continuously differentiable elastic coefficients behaves like homogeneous medium with respect to singularity, in a nonhomogeneous medium with piecewise constant elastic coefficients the behaviour is quite different; for example, stress field around the crack tip terminating at the interface shows a behaviour of the form r^{α}, where r is the distance from the crack tip and $-1 < \alpha < 0$ (Erdogan [4]).

In solid mechanics, as the idea of homogeneity of the medium is not always adequate and since there are plenty of nonhomogeneous materials, investigations of crack problems in nonhomogeneous media are necessary as well as interesting. It is true that nonhomogeneity of a medium depends on many parameters, not all of which may be known, so that various problems of solid mechanics in nonhomogeneous media are studied by considering suitable models with specific types of nonhomogeneity. Various models have been considered in the literature to discuss the crack problems. Models with continuous variation of elastic coefficients in more than one direction have also been considered (Dhaliwal and Singh [3], Chaudhuri and Ray [1]).

In the paper [2], DELALE and ERDOGAN discussed the crack problem for a nonhomogeneous plane assuming nonhomogeneity of Young's modulus and taking a uniform Poisson's ratio. They posed the problem by assuming $E = E_0 \exp(\beta x + \gamma y)$, but presented the solution only for $\gamma = 0$.

The aim of the present investigation is to solve the same problem as that considered by Delale and Erdogan but with $\gamma \neq 0$. The problem has been solved and the results are presented analytically in the form of integrals, and also graphically. Results of Delale and Erdogan are recovered as a special case of our discussion.

2. FORMULATION OF THE CRACK PROBLEM

We shall consider the plane elasticity problem for a nonhomogeneous solid in which the Poisson's ratio ν is constant and the Young's modulus E is a function of x and y. The stresses are given by

$$\sigma_{xx} = \frac{\partial^2 F}{\partial y^2}, \qquad \sigma_{yy} = \frac{\partial^2 F}{\partial x^2}, \qquad \sigma_{xy} = -\frac{\partial^2 F}{\partial x \partial y}, \tag{2.1}$$

where $F(x, y)$ is the Airy stress function.

For the plane elasticity problem the compatibility equation for generalized plane stress, by using Eq. (2.1) and Hooke's law, is given by

$$\begin{aligned} E^2 \nabla^4 F - 2E \left(\frac{\partial E}{\partial x}\frac{\partial}{\partial x} + \frac{\partial E}{\partial y}\frac{\partial}{\partial y} \right) \nabla^2 F \\ +2(1+\nu)\left(2\frac{\partial E}{\partial x}\frac{\partial E}{\partial y} - E\frac{\partial^2 E}{\partial x \partial y}\right)\frac{\partial^2 F}{\partial x \partial y} \\ + \left[2\left(\frac{\partial E}{\partial x}\right)^2 - 2\nu\left(\frac{\partial E}{\partial y}\right)^2 - E\frac{\partial^2 E}{\partial x^2} + \nu E \frac{\partial^2 E}{\partial y^2}\right]\frac{\partial^2 F}{\partial x^2} \\ + \left[2\left(\frac{\partial E}{\partial y}\right)^2 - 2\nu\left(\frac{\partial E}{\partial x}\right)^2 - E\frac{\partial^2 E}{\partial y^2} + \nu E \frac{\partial^2 E}{\partial x^2}\right]\frac{\partial^2 F}{\partial y^2} = 0. \end{aligned} \tag{2.2}$$

In the plane strain case, Eq. (2.2) is the same except for E being replaced by $E/(1-\nu^2)$ and ν by $\nu/(1-\nu)$.

In our problem we consider a line crack of length $2a$ occupying the position $|x| < a$, $y = 0$ in a nonhomogeneous medium, in which the Young's modulus varies exponentially with x and y in the form

$$E = E_0 \exp(\beta x + \gamma |y|), \tag{2.3}$$

while the Poisson's ratio is constant. E_0, β, γ appearing in Eq. (2.3) are constants.

If we further assume that the load applied to the medium is symmetrical with respect to $y = 0$, then from the conditions of symmetry of the material nonhomogeneity and of the applied load with respect to $y = 0$, it would be sufficient to consider the solution of the problem for the medium $-\infty < x < \infty$, $y \geq 0$ only.

Hence the nonhomogeneity of the medium may be taken in the form

$$E = E_0 \exp(\beta x + \gamma y), \qquad -\infty < x < \infty, \quad y \geq 0. \tag{2.4}$$

By using Eq. (2.4), Eq. (2.2) reduces to

$$\begin{aligned}\nabla^4 F - 2\beta\left(\frac{\partial^3 F}{\partial x^3} + \frac{\partial^3 F}{\partial x \partial y^2}\right) - 2\gamma\left(\frac{\partial^3 F}{\partial y \partial x^2} + \frac{\partial^3 F}{\partial y^3}\right)& \\ + \left(\beta^2 - \nu\gamma^2\right)\frac{\partial^2 F}{\partial x^2} + 2(1+\nu)\beta\gamma\frac{\partial^2 F}{\partial x \partial y} + \left(\gamma^2 - \nu\beta^2\right)\frac{\partial^2 F}{\partial y^2} &= 0.\end{aligned} \tag{2.5}$$

It is noted that Eq. (2.5) reduces to the standard biharmonic equation only when $\beta = \gamma = 0$.

Considering the solution of Eq. (2.5) in the form

$$F(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(y,\alpha)e^{-ix\alpha}\,d\alpha \qquad (-\infty < x < \infty, \quad y > 0) \tag{2.6}$$

we obtain

$$\begin{aligned}\frac{d^4 f}{dy^4} - 2\gamma\frac{d^3 f}{dy^3} + \left(2i\alpha\beta - 2\alpha^2 + \gamma^2 - \nu\beta^2\right)\frac{d^2 f}{dy^2}& \\ +2\left\{\alpha^2\gamma - i(1+\nu)\alpha\beta\gamma\right\}\frac{df}{dy} + \left(\alpha^4 - 2i\alpha^3\beta - \alpha^2\beta^2 + \nu\alpha^2\gamma^2\right)f &= 0.\end{aligned} \tag{2.7}$$

If we take the solution of Eq. (2.7) in the form $f = e^{-my}$, then the auxiliary equation is given by

$$\begin{aligned}m^4 + 2\gamma m^3 + \left(2i\alpha\beta - 2\alpha^2 + \gamma^2 - \nu\beta^2\right)m^2& \\ -2\left(\alpha^2\gamma - i\alpha\beta\gamma - i\nu\alpha\beta\gamma\right)m + \left(\alpha^4 - 2i\alpha^3\beta - \alpha^2\beta^2 + \nu\alpha^2\gamma^2\right) &= 0.\end{aligned} \tag{2.8}$$

Equation (2.8) yields the following solutions for m:

$$\begin{aligned} m_1 &= \left(\delta_1 + \beta\sqrt{\nu} - \gamma\right)/2, \\ m_2 &= \left(\delta_2 - \beta\sqrt{\nu} - \gamma\right)/2, \\ m_3 &= -\left(\delta_1 - \beta\sqrt{\nu} + \gamma\right)/2, \\ m_4 &= -\left(\delta_2 + \beta\sqrt{\nu} + \gamma\right)/2, \end{aligned} \tag{2.9}$$

where

$$(2.10)\qquad \begin{aligned} \delta_1 &= \left[(\beta\sqrt{\nu}-\gamma)^2 + 4\alpha^2 - 4i\alpha\left(\beta-\gamma\sqrt{\nu}\right)\right]^{1/2}, \\ \delta_2 &= \left[(\beta\sqrt{\nu}+\gamma)^2 + 4\alpha^2 - 4i\alpha\left(\beta+\gamma\sqrt{\nu}\right)\right]^{1/2}, \end{aligned}$$

$\mathrm{Re}\,(m_1) > 0$, $\mathrm{Re}\,(m_2) > 0$. So the appropriate solution of Eq. (2.7) can be expressed in the form

$$(2.11)\qquad f(y,\alpha) = A_1(\alpha)e^{-m_1 y} + A_2(\alpha)e^{-m_2 y} \qquad (0 < y < \infty).$$

Using Eq. (2.6) and Eq. (2.11), we obtain from Eq. (2.1)

$$(2.12)\qquad \sigma_{xx}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\sum_{j=1}^{2} A_j m_j^2 e^{-m_j y} e^{-ix\alpha}\,d\alpha,$$

$$(2.13)\qquad \sigma_{yy}(x,y) = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\alpha^2\sum_{j=1}^{2} A_j e^{-m_j y} e^{-ix\alpha}\,d\alpha,$$

$$(2.14)\qquad \sigma_{xy}(x,y) = -\frac{i}{2\pi}\int_{-\infty}^{\infty}\alpha\sum_{j=1}^{2} A_j m_j e^{-m_j y} e^{-ix\alpha}\,d\alpha.$$

For the plane $y > 0$, the functions A_1 and A_2 are known from the two boundary conditions at $y = 0$, $-\infty < x < \infty$.

Let us suppose that the half-plane $y > 0$ is subjected to tractions

$$(2.15)\qquad \sigma_{yy}(x,0) = \sigma(x), \qquad \sigma_{xy}(x,0) = \tau(x) \qquad (-\infty < x < \infty)$$

and is kept in equilibrium by a resultant force applied to the medium at infinity, which is collinear with a force defined by the following components:

$$(2.16)\qquad P_x = \int_{-\infty}^{\infty}\tau(x)\,dx, \qquad P_y = \int_{-\infty}^{\infty}\sigma(x)\,dx.$$

From Eqs. (2.13), (2.14), (2.15) and using the Fourier transform we obtain

$$(2.17)\qquad A_1(\alpha) = \frac{1}{m_1 - m_2}\left[\frac{m_2 Q_1}{\alpha^2} + \frac{iQ_2}{\alpha}\right],$$

$$(2.18)\qquad A_2(\alpha) = \frac{1}{m_2 - m_1}\left[\frac{iQ_2}{\alpha} + \frac{m_1 Q_1}{\alpha^2}\right],$$

where

$$(2.19)\qquad Q_1(\alpha) = \int_{-\infty}^{\infty}\sigma(x)e^{i\alpha x}\,dx, \qquad Q_2(\alpha) = \int_{-\infty}^{\infty}\tau(x)e^{i\alpha x}\,dx.$$

3. Integral equation

We shall consider the original cracked solid to be loaded symmetrically in such a way that

$$\sigma_{xy}(x,0) = 0, \qquad -\infty < x < \infty. \tag{3.1}$$

In the perturbation problem, in addition to Eq. (3.1), we have the mixed boundary conditions

$$\sigma_{yy}(x,0+) = p(x), \qquad -a < x < a, \tag{3.2}$$

$$v(x,0) = 0, \qquad a < |x| < \infty, \tag{3.3}$$

where $p(x)$ is a known function and v is the displacement component in the y direction.

From Eqs. (2.14) and (3.1), we obtain

$$m_1 A_1 + m_2 A_2 = 0. \tag{3.4}$$

We consider a new unknown function $g(x)$

$$g(x) = \frac{\partial}{\partial x} v(x, 0+). \tag{3.5}$$

From Eqs. (3.3) and (3.5), we obtain

$$g(x) = 0 \qquad \text{for} \qquad |x| > a$$

and

$$\int_{-a}^{a} g(x)\, dx = 0. \tag{3.6}$$

By using Hooke's law, from Eqs. (2.12) and (2.13) we obtain

$$\frac{\partial}{\partial x} v(x,y) = -\frac{1}{2\pi} \frac{1}{E(x,y)} \int_{-\infty}^{\infty} (\beta + i\alpha) \left[\frac{A_1}{m_1 + \gamma} \left(\alpha^2 + \nu m_1^2\right) e^{-m_1 y} + \frac{A_2}{m_2 + \gamma} \left(\alpha^2 + \nu m_2^2\right) e^{-m_2 y} \right] e^{-ix\alpha}\, d\alpha \qquad (y > 0). \tag{3.7}$$

From Eqs. (3.5) and (3.7), we obtain

$$-\frac{(\beta + i\alpha)}{E_0} \left[\left(\alpha^2 + \nu m_1^2\right) \frac{A_1}{m_1 + \gamma} + \left(\alpha^2 + \nu m_2^2\right) \frac{A_2}{m_2 + \gamma} \right] = \int_{-a}^{a} g(t) e^{(\beta + i\alpha)t}\, dt. \tag{3.8}$$

Solving A_1 and A_2 from Eqs. (3.4) and (3.8), we get

$$(3.9)\qquad A_1(\alpha) = \frac{E_0(m_1+\gamma)(m_2+\gamma)m_2}{(\beta+i\alpha)(m_1-m_2)\left[\alpha^2(m_1+m_2)+\gamma(\alpha^2-m_1m_2\nu)\right]} \times \int_{-a}^{a} g(t)e^{(\beta+i\alpha)t}\,dt = -\frac{m_2}{m_1}A_2(\alpha).$$

Using Eqs. (3.9) and (2.13), we obtain the integral equation to determine $g(x)$ from Eq. (3.2) as follows:

$$(3.10)\qquad \lim_{y\to 0+}\frac{1}{2\pi}\int_{-a}^{a} g(t)e^{\beta t}\int_{-\infty}^{\infty}\frac{E_0(m_1+\gamma)(m_2+\gamma)\alpha^2}{(\beta+i\alpha)(m_1-m_2)[\alpha^2(m_1+m_2)+\gamma(\alpha^2-m_1m_2\nu)]} \times\left(m_1e^{-m_2y}-m_2e^{-m_1y}\right)e^{i(t-x)\alpha}\,d\alpha = p(x), \qquad |x|<a.$$

To separate a possible singular part of the kernel in Eq. (3.10), we examine the asymptotic behaviour of the inner integral. From Eq. (2.9) we see that for $|\alpha|\to\infty$, $m_1\to|\alpha|$ and $m_2\to|\alpha|$.

The inner integral in Eq. (3.10) can be expressed as

$$(3.11)\qquad h(x,y,t) = \int_{-\infty}^{\infty} K(y,\alpha)e^{i(t-x)\alpha}\,d\alpha.$$

Since any possible singular part of h must be due to the behaviour of K at $|\alpha|\to\infty$, so we may write Eq. (3.11) as

$$(3.12)\qquad h(x,y,t) = \int_{-\infty}^{\infty}\Big[K(y,\alpha)-K_\infty(y,\alpha)\Big]e^{i(t-x)\alpha}\,d\alpha + \int_{-\infty}^{\infty} K_\infty(y,\alpha)e^{i(t-x)\alpha}\,d\alpha,$$

where K_∞ is the asymptotic value of $K(y,\alpha)$ for large values of $|\alpha|$.

It can be easily shown that

$$(3.13)\qquad K_\infty(y,\alpha) = \frac{E_0(|\alpha|+\gamma)e^{-|\alpha|y}}{2i\alpha}.$$

The first integral in Eq. (3.12) is uniformly convergent. Therefore, when it is substituted in Eq. (3.10), the limit can be put under the integral sign. The second integral in Eq. (3.12), by using Eq. (3.13), can be expressed as

$$(3.14)\qquad \int_{-\infty}^{\infty} \frac{E_0(|\alpha|+\gamma)e^{-|\alpha|y}}{2i\alpha}\Big[\cos(t-x)\alpha + i\sin(t-x)\alpha\Big]\,d\alpha = \frac{E_0(t-x)}{(t-x)^2+y^2} + E_0\gamma \operatorname{arc\,tg}\frac{(t-x)}{y}.$$

Let

$$(3.15)\qquad M(\alpha) = \frac{\alpha^2(m_1+\gamma)(m_2+\gamma)e^{i(t-x)\alpha}}{(\beta+i\alpha)[\alpha^2(m_1+m_2)+\gamma(\alpha^2-m_1m_2\nu)]}.$$

Putting $y=0$ in the first integral in Eq. (3.12), we obtain after some calculations

$$(3.16)\qquad \lim_{y\to 0} h(x,y,t) = E_0\int_0^{\infty}\Big[M(\alpha)+M(-\alpha)-\frac{\alpha+\gamma}{\alpha}\sin(t-x)\alpha\Big]\,d\alpha + \lim_{y\to 0}\Big[\frac{E_0(t-x)}{(t-x)^2+y^2} + E_0\gamma\operatorname{arc\,tg}\frac{(t-x)}{y}\Big].$$

Substituting from Eq. (3.16) into Eq. (3.10) we obtain

$$(3.17)\qquad \frac{1}{\pi}\int_{-a}^{a}\Big[\frac{e^{\beta t}}{t-x} + R(x,t)\Big]g(t)\,dt = \frac{1+\zeta}{4\mu_0}p(x), \qquad |x|<a,$$

where the Fredholm kernel is given by

$$(3.18)\qquad R(x,t) = e^{\beta t}\Big[\frac{\gamma\pi}{2} + \int_0^{\infty}\Big\{M(\alpha)+M(-\alpha)-\frac{\alpha+\gamma}{\alpha}\sin(t-x)\alpha\Big\}\,d\alpha\Big],$$

and where $E_0/2$ is replaced by $4\mu_0/(1+\zeta)$ in order to cover both the generalized plane stress and plane strain problems. Here μ_0 is the shear modulus at $x=0$, i.e. $\mu_0 = E_0/2(1+\nu)$, $\zeta = 3-4\nu$ for plane strain and $\zeta = (3-\nu)/(1+\nu)$ for generalized plane stress.

We note that if $\gamma = 0$, then all the results reduce to those obtained in [2]. It is also noted that if $\beta = \gamma = 0$ then $K = K_\infty$, $R(x,t) = 0$ and hence Eq. (3.17) reduces to the known integral equation of the simple (Mode I) crack problem for homogeneous plane.

For numerical solution, the interval $(-a,a)$ is normalized by defining

$$(3.19)\qquad \begin{gathered} s = t/a, \qquad r = x/a, \qquad \phi(s) = g(t), \qquad n(r,s) = R(x,t), \\ q(r) = p(x), \qquad -1<(r,s)<1, \qquad -a<(x,t)<a. \end{gathered}$$

So by using relations (3.19), Eqs. (3.17) and (3.6) are expressed as

$$\frac{1}{\pi}\int_{-1}^{1}\left[\frac{e^{a\beta s}}{s-r}+n(r,s)\right]\phi(s)\,ds=\frac{1+\zeta}{4\mu_0}q(r) \qquad (-1<r<1), \tag{3.20}$$

$$\int_{-1}^{1}\phi(s)\,ds=0. \tag{3.21}$$

4. Stress intensity factors

Since the index of the singular integral equation (3.20) is +1, its solution is taken in the form

$$e^{a\beta s}\phi(s)=\frac{G(s)}{\sqrt{1-s^2}}, \qquad -1<s<1, \tag{4.1}$$

where $G(s)$ is a bounded function.

The unknown function G can be determined from Eqs. (3.20) and (3.21) to any desired degree of accuracy.

It is observed that the left-hand side of Eq. (3.17) gives $\sigma_{yy}(x,0)$ for $|x|>a$ as well as $|x|<a$; by means of a simple asymptotic analysis, the Mode I stress intensity factors at the crack tips defined by

$$k_1(a)=\lim_{x\to a}\sqrt{2(x-a)}\sigma_{yy}(x,0), \tag{4.2}$$

$$k_1(-a)=\lim_{x\to -a}\sqrt{2(-x-a)}\sigma_{yy}(x,0), \tag{4.3}$$

may be expressed by

$$k_1(a)=-\frac{4}{1+\zeta}\mu_0 G(1)\sqrt{a}, \tag{4.4}$$

$$k_1(-a)=\frac{4}{1+\zeta}\mu_0 G(-1)\sqrt{a}. \tag{4.5}$$

After obtaining $G(s)$, the crack surface displacement can be calculated from Eqs. (3.5) and (3.19) as

$$\frac{v(x)}{a}=\int_{-1}^{x/a}\frac{G(s)}{\sqrt{1-s^2}}e^{-a\beta s}\,ds. \tag{4.6}$$

It is also noted that the structure of the integral equation (3.17) is the same as that for a homogeneous medium, namely its kernel has a simple Cauchy singularity. Thus its solution, and consequently the stress state around the crack tip, would have the conventional square root singularity.

5. Numerical results and discussion

To get some idea about the effects of nonhomogeneity of the type considered in our discussion, numerical computations have been done in some particular cases. In order to compare our results with those of [2] and to see the effects of additional nonhomogeneity, we have considered, as in [2], two types of loadings. In the first case we assume that the loading is such that in the uncracked medium

$$\varepsilon_{yy}(x,0) = \varepsilon_0 + \varepsilon_1\left(\frac{x}{a}\right), \qquad \varepsilon_{xy}(x,0) = 0, \tag{5.1}$$
$$\sigma_{xx}(x,0) = 0.$$

Then it readily follows that on the crack surface

$$\sigma_{yy}(x,0) = p(x) = -\varepsilon_0 E_0 e^{\beta x} - \varepsilon_1 E_0\left(\frac{x}{a}\right)e^{\beta x}, \tag{5.2}$$
$$\sigma_{yx}(x,0) = 0, \qquad |x| < a.$$

In the second type of loading we assume

$$p(x) = p_0 - p_1(x/a). \tag{5.3}$$

The stress intensity factors (SIF) at the crack tips in these two types of loadings have been computed from Eqs. (4.4) and (4.5) and the results are given in Table 1–4 for various values of $a\beta$ and $a\gamma$. Although Poisson's ratio ν has effects on the SIF, we have considered only $\nu = 0.3$ in our computations of the SIF. In each table the columns corresponding to $a\gamma = 0$ give results which are almost identical with those of Delale and Erdogan's paper. As in [2], we have considered only one nonzero parameter at a time out of four parameters ε_0, ε_1, p_0, p_1 in Eqs. (5.2) and (5.3) for

Table 1. Normalized SIF for the case of generalized plane stress ($\nu = 0.3$) for $p(x) = -\varepsilon_0 E_0 e^{\beta x}$.

	$k_1(-a)/\varepsilon_0 E_0\sqrt{a}$					$k_1(a)/\varepsilon_0 E_0\sqrt{a}$				
$a\beta$ \ $a\gamma$	0	0.25	0.50	0.75	1.00	0	0.25	0.50	0.75	1.00
0	1.0	0.926	0.836	0.754	0.684	1.0	0.982	0.940	0.893	0.848
0.2	0.853	0.798	0.716	0.638	0.574	1.162	1.053	0.948	0.872	0.806
0.4	0.722	0.671	0.599	0.537	0.481	1.344	1.191	1.024	0.888	0.810
0.6	0.608	0.570	0.498	0.445	0.400	1.552	1.363	1.155	0.994	0.869
0.8	0.511	0.491	0.415	0.369	0.332	1.793	1.576	1.322	1.139	0.996
1.0	0.429	0.434	0.349	0.306	0.275	2.075	1.839	1.530	1.323	1.164

the purpose of observing the effects of nonhomogeneity, although the joint effect may be obtained by superposing the individual effects. From Tables 1 – 3 it is clear that for fixed $a\beta$, the SIF decreases with the increase of $a\gamma$ which physically indicates smaller SIF for stiffer materials. The behaviour is slightly different at the right-hand end of the crack, what is observed in Table 4. In the computation of the function $G(s)$ in (4.1) we have adopted the Gauss - Tchebycheff integration technique.

Table 2. Normalized SIF for the case of generalized plane stress ($\nu = 0.3$) for $p(x) = -\varepsilon_1 E_0 (x/a) e^{\beta x}$.

	$k_1(-a)/\varepsilon_1 E_0\sqrt{a}$					$k_1(a)/\varepsilon_1 E_0\sqrt{a}$				
$a\beta \backslash a\gamma$	0	0.25	0.50	0.75	1.00	0	0.25	0.50	0.75	1.00
0	−0.5	−0.497	−0.490	−0.480	−0.468	0.5	0.497	0.492	0.486	0.480
0.2	−0.408	−0.404	−0.399	−0.391	−0.382	0.611	0.608	0.601	0.594	0.586
0.4	−0.332	−0.328	−0.323	−0.318	−0.311	0.744	0.737	0.721	0.703	0.690
0.6	−0.269	−0.264	−0.262	−0.258	−0.252	0.904	0.890	0.864	0.836	0.806
0.8	−0.218	−0.211	−0.212	−0.209	−0.205	1.094	1.073	1.032	0.993	0.954
1.0	−0.176	−0.164	−0.171	−0.170	−0.166	1.322	1.293	1.232	1.180	1.129

Table 3. Normalized SIF for the case of generalized plane stress ($\nu = 0.3$) for $p(x) = -p_0$.

	$k_1(-a)/p_0\sqrt{a}$					$k_1(a)/p_0\sqrt{a}$				
$a\beta \backslash a\gamma$	0	0.25	0.50	0.75	1.00	0	0.25	0.50	0.75	1.00
0	1.0	0.926	0.836	0.754	0.684	1.0	0.982	0.940	0.893	0.848
0.2	0.943	0.887	0.804	0.724	0.658	1.051	0.942	0.838	0.763	0.698
0.4	0.885	0.832	0.757	0.690	0.631	1.097	0.944	0.780	0.649	0.575
0.6	0.830	0.787	0.711	0.652	0.602	1.140	0.952	0.750	0.599	0.484
0.8	0.780	0.752	0.671	0.618	0.574	1.181	0.965	0.723	0.557	0.434
1.0	0.735	0.727	0.638	0.587	0.549	1.221	0.985	0.699	0.519	0.392

Table 4. Normalized SIF for the case of generalized plane stress ($\nu = 0.3$) for $p(x) = -p_1(x/a)$.

	$k_1(-a)/p_1\sqrt{a}$					$k_1(a)/p_1\sqrt{a}$				
$a\beta \backslash a\gamma$	0	0.25	0.50	0.75	1.00	0	0.25	0.50	0.75	1.00
0	−0.5	−0.497	−0.490	−0.480	−0.468	0.5	0.497	0.492	0.486	0.480
0.2	−0.498	−0.491	−0.482	−0.470	−0.457	0.500	0.502	0.501	0.498	0.493
0.4	−0.494	−0.484	−0.472	−0.460	−0.447	0.499	0.507	0.508	0.503	0.499
0.6	−0.488	−0.476	−0.463	−0.449	−0.436	0.498	0.512	0.516	0.512	0.501
0.8	−0.480	−0.468	−0.453	−0.439	−0.425	0.496	0.516	0.524	0.521	0.509
1.0	−0.472	−0.459	−0.443	−0.429	−0.416	0.493	0.520	0.533	0.529	0.516

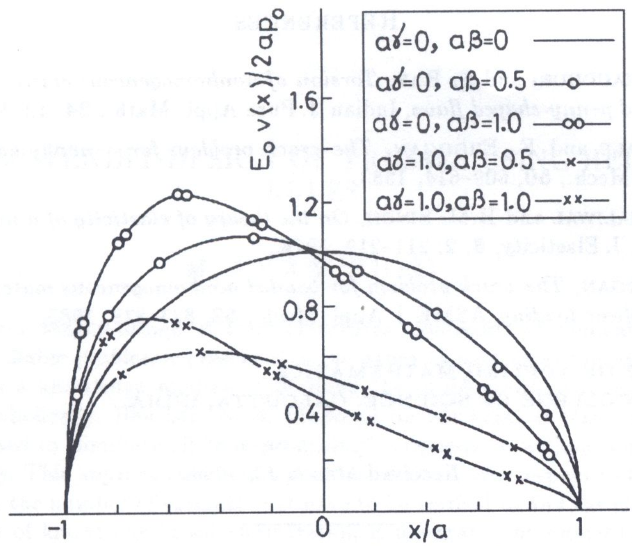

FIG. 1. Crack surface displacement for various x/a under uniform pressure p_0 applied to the crack surface; $\nu = 0.5$, plane stress condition. First type of loading.

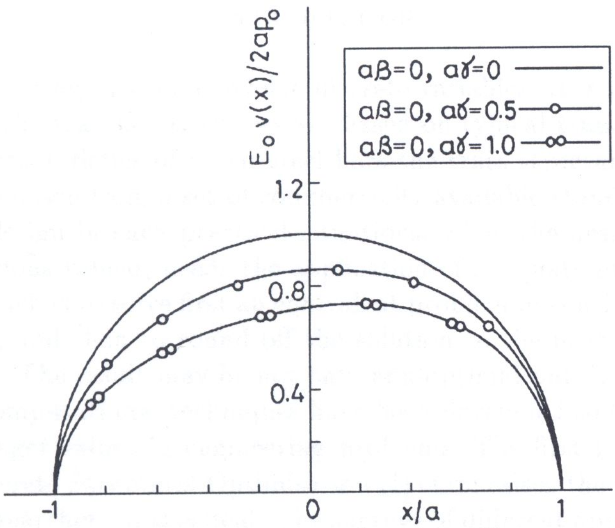

FIG. 2. Crack surface displacement for various x/a under uniform pressure p_0 applied to the crack surface; $\nu = 0.5$, plane stress condition. Second type of loading.

Significant effects of nonhomogeneity are also noticeable in the crack surface displacement. The results are given in Figs. 1 and 2. The effects of nonhomogeneity when E varies in one or both the directions are clearly shown in the figures which indicate, as expected, less stiff material exhibiting greater displacement.

REFERENCES

1. P.K. CHAUDHURI and S. RAY, *Torsion of nonhomogeneous elastic half-space with embedded penny-shaped flaws*, Indian J. Pure Appl. Math., **24**, 12, 801–815, 1993.

2. F. DELALE and F. ERDOGAN, *The crack problem for a nonhomogeneous plane*, J. Appl. Mech., **50**, 609–614, 1983.

3. R.S. DHALIWAL and B.M. SINGH, *On the theory of elasticity of a nonhomogeneous medium*, J. Elasticity, **8**, 2, 211–219, 1978.

4. F. ERDOGAN, *The crack problem for bonded nonhomogeneous materials under antiplane shear loading*, ASME J. Appl. Mech., **52**, 823–828, 1985.

DEPARTMENT OF APPLIED MATHEMATICS
UNIVERSITY COLLEGE OF SCIENCE, CALCUTTA, INDIA.

Received March 13, 1996.